Balanced Three Phase Power Systems

Topics

Purpose

In balanced systems net instantaneous power is constant

Turbines driving a generator see constant counter torque corresponding to real power

Motors produce constant torque

Reactive power ‘exists’ in phases of the electric circuit but mechanical power is constant and corresponds to real power

Schematics, Nomenclature, Definitions

Wye – 3 and 4 wire

Delta

Conventions – Unless otherwise stated

Voltages are given Line-Line

Currents are line currents

Power is total three phase

Stuff is modeled as wye connected

Analysis -- Balanced Systems

Use per-phase model (also called single phase or line-neutral model)

Relationships between three phase voltages, currents and power

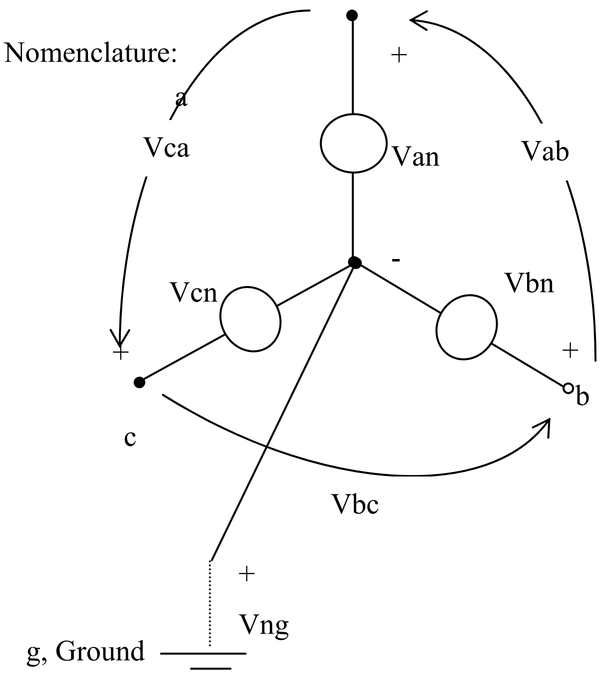
Instantaneous and Complex Power

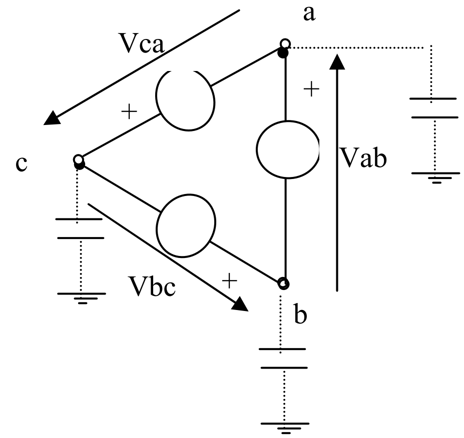
Constant Power Loads

Problem Solving

Schematics, Nomenclature, Definitions

Nomenclature:





The source shown on the left, above, is a WYE connected source

a., b, c denote the three phase terminals

n is the neutral which may or may not be connected to ground g.

Ground g is typically the equipotential surface of earth with good contact achieved through driven ground rods, metal water pipes or more elaborate structures

Voltages Van, Vbn and Vcn are LINE-NEUTRAL voltages

These are also popularly called PHASE voltages

Voltages Vab, Vbc , Vca are LINE-LINE voltages

Also called LINE voltages

Voltage Vng is neutral to ground voltage

We can also define Voltages Vag, Vbg, Vcg – these are line-ground voltages. Only Vag is marked in the figure.

The source to the right is a DELTA connected source. It has no connection to ground except through stray capacitance. Only line-line voltages are defined.

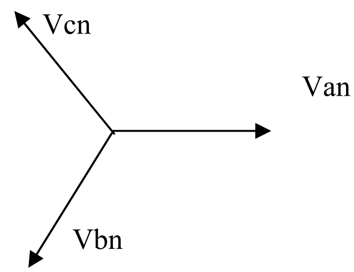
Often, each leg in a delta is called a 'phase'

The voltage is also called a 'phase voltage'

BALANCED POSITIVE SEQUENCE VOLTAGE in a wye-connected source is defined as a set of three voltages, say line-neutral, with equal magnitudes and with phase b lagging phase a by 120deg and phase c lagging phase b by 120 deg.

Van = VLN /$\underline{0^{o}}$ Vbn = VLN /$\underline{-120^{o}}$ Vcn = VLN /$\underline{-240}$ (or Vcn = VLN/$\underline{-120^{o}}$)

These can be shown in a phasor diagram form



Balanced NEGATIVE Sequence voltages are defined as

Van = VLN /$\underline{0^{o}}$ Vbn = VLN /$\underline{-120^{o}}$ Vcn = VLN /$\underline{-240}$ (or Vcn = VLN/$\underline{+120^{o}}$)

These can be shown in a phasor diagram form.
BALANCED NEGATIVE SEQUENCE VOLTAGE in a wye-connected source is defined as a set of three voltages, say line-neutral,

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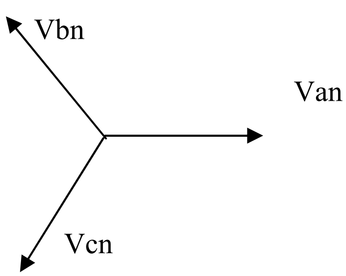


A positive sequence source can be turned into negative sequence by swapping a pair of leads

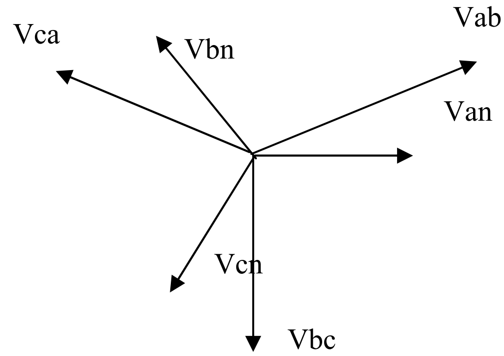
Sequence determines the direction of rotation of Polyphase ac motors

We will assume BPS unless otherwise stated

LINE-LINE and LINE –NEUTRAL VOLTAGES FOR BALANCED POSITIVE SEQUENCE

It is always true (think KVL) that Vab= Van-Vbn. In the case of a delta imagine ‘n’ to be a fictitious neutral point. Similarly, Vbc=Vbn-Vcn and Vca=Vcn-Van.

We can then show that for BPS

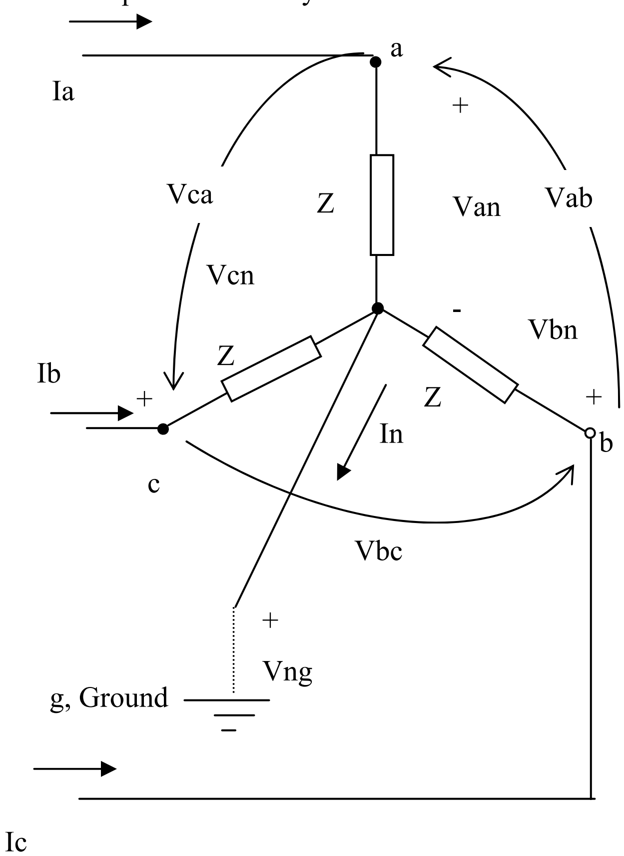


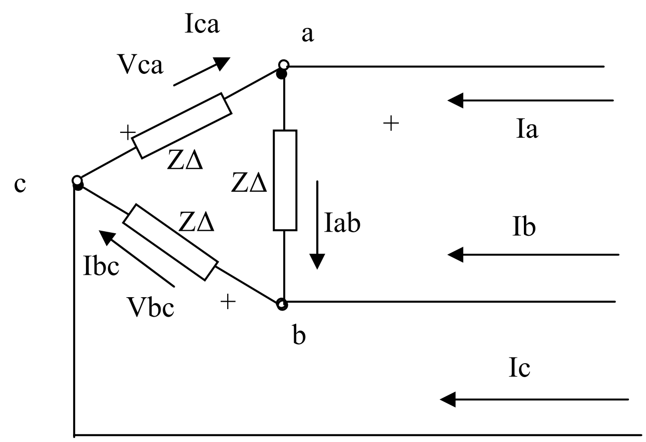
$Vab=\sqrt{3}\ Van/\underline{30^o}$, $Vbc=\sqrt{3}\ Vbn/\underline{30^o}$, $Vab=\sqrt{3}\ Vcn/\underline{30^o}$

The magnitude of L-L voltage $VLL= Vab=\sqrt{3}\ VLN$

These relations apply to delta sources as well, with 'n' to be a fictitious neutral point. Thus a balanced delta source is equivalent to an ungrounded wye source.

BALANCED THREE PHASE LOADS consist of three equal impedances in a wye or delta connection. We will also model model three phase loads as constant KVA loads in which case the complex power S consumed in each phase will be assumed to be equal. Note that a delta-load is equivalent to a wye load with Z Δ = 3Z ans S Δ = S





THREE PHASE CURRENTS

In the loads above the wires connecting to the load terminals are the LINES

Ia, Ib, Ic are called LINE CURRENTS

-They are called phase currents for wye

In is called the neutral current Note: In=Ia+Ib+Ic

Iab, Ibc, Ica in the Delta load are delta currents

--These are often called phase currents for a delta

BLANCED POSITIVE SEQUENCE CURRENTS are defined as

Ia = IL /$\underline{0^o}$ Ib = IL /$\underline{-120^o}$ Ic= IL /$\underline{-240}$

Can show that for a delta **load** Iab=(Ia/√3)/$\underline{30^o}$, Ibc=(Ib/√3)/$\underline{30^o}$, Ica=(Ic/√3)/$\underline{30^o}$ for BPS

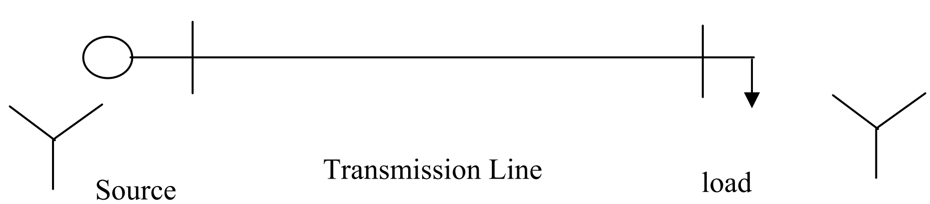
BLANCED 3 PHASE SYSTEMS ANALYSIS

We need only consider wye connections since balanced delta sources and loads can be converted to wye

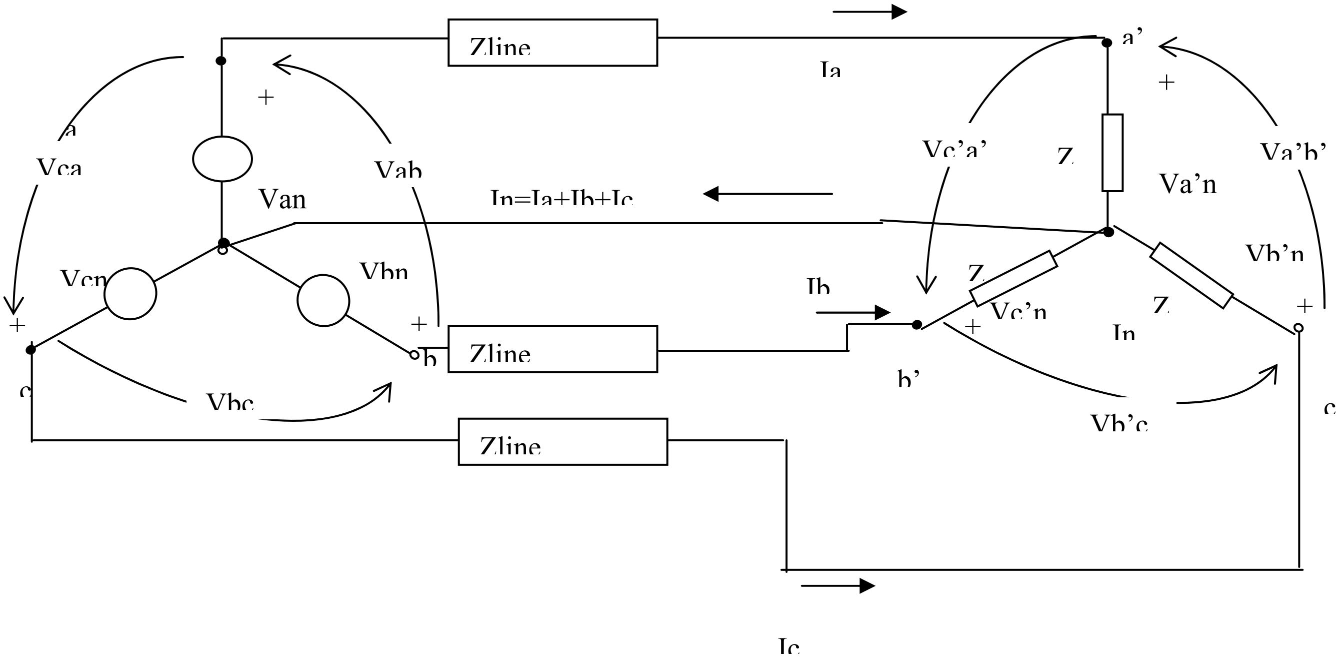
A balanced, positive sequence, 60 Hz, three-phase source supplies a balanced wye connected load through a three-phase transmission line. The neutral of the source is connected to that of the load through a wire with negligible impedance. The load impedance is 10 ohm/phase and line impedance is 1+j1 ohms per phase.

Find all line currents, line-neutral load voltages, line-line load voltages.

One line diagram



Three-line diagram



Remember, by convention, the given source voltage of 208 V is a LINE-LINE, RMS voltag

So $Vab := 208 \cdot e^{j \cdot 0deg}$

Since source is BPS

$Vab := 208 \cdot e^{j \cdot 0deg}$ $\quad Vbc := 208 \cdot e^{j \cdot -120deg}$ $\quad Vca := 208 \cdot e^{j \cdot 120deg}$

Then $Van := \frac{208}{\sqrt{3}} \cdot e^{j \cdot -30deg}$ $\quad Vbn := \frac{208}{\sqrt{3}} \cdot e^{j \cdot -150deg}$ $\quad Vcn := \frac{208}{\sqrt{3}} \cdot e^{j \cdot 90deg}$ V

It is however convenient to let Van be the voltage reference so let

$Vab := 208 \cdot e^{j \cdot 30deg}$ $\quad Vbc := 208 \cdot e^{j \cdot -90deg}$ $\quad Vca := 208 \cdot e^{j \cdot 150deg}$

Then $Van := \frac{208}{\sqrt{3}} \cdot e^{j \cdot 0deg}$ $\quad Vbn := \frac{208}{\sqrt{3}} \cdot e^{j \cdot -120deg}$ $\quad Vcn := \frac{208}{\sqrt{3}} \cdot e^{j \cdot 120deg}$

Mesh analysis of the loop consisting of the phase source, line, load and neutral gives

Van= Ia Zline+ Ia Z

$Zline := 1 + j \qquad Z := 10$

$Ia := \frac{Van}{Z + Zline} \quad Ia = 10.828 - 0.984i \quad |Ia| = 10.872 \quad arg(Ia) = -5.194\,deg$

$Va'n := Van - Ia \cdot Zline \quad |Va'n| = 108.723 \quad arg(Va'n) = -5.194\,deg$

Similarly

$Ib := \frac{Vbn}{Z + Zline} \quad Ia = 10.828 - 0.984i \quad |Ia| = 10.872 \quad arg(Ia) = -5.194\,deg$

$Vb'n := Vbn - Ib \cdot Zline \quad |Vb'n| = 108.723 \quad arg(Vb'n) = -125.194\,deg$

$Ic := \frac{Vcn}{Z + Zline} \quad Ic = -4.561 + 9.869i \quad |Ic| = 10.872 \quad arg(Ic) = 114.806\,deg$

$Vc'n := Vcn - Ic \cdot Zline \quad |Vc'n| = 108.723 \quad arg(Vc'n) = 114.806\,deg$

Notice that load current and voltage are Balanced Positive Sequence!!!!!

If sources, loads and network component are balance all voltages and currents are balanced

Therefore we could simply analyze phase ‘a’ and then find other voltages and currents by using the basic property of balanced voltages and currents.

BALANCED, THREE PHASE SYSTEMS ARE ANALYZED USING A PER PHASE OR LINE_NEUTRAL MODEL!!!!!!!!!

Finally, note that, since Ia, Ib, Ic are balanced, In=Ia+Ib+Ic=0; our conclusion thus holds even if the neutrals are not connected together.

GENERAL ANALYSIS AND UNBALANCED SYSTEMS

In the general case one would simply write KVL and KCL to solve the problem. In our example assume that the neutrals are not connected but that the circuit is balanced.

Van -Vbn = Vab = Ia Zline+Ia Zload -Ib Zload -IbZline
Vbn -Vcn = Vab = Ib Zline+Ib Zload -Ic Zload -IcZline
Vcn -Van = Vab = Ic Zline+Ic Zload -Ia Zload -IaZline

Unfortunately, these equations are dependent so we need one more independent equation. KCL at the neutral (remeember, neutrals are disconnected for this example)

Ia+Ib+Ic=0

So, we have

Vab=208/<u>30</u>= Ia(Zline+Zload)+Ib(-Zline-Zload)+Ic*0
Vbc=208/<u>-90</u>= Ic*0+Ib(Zline+Zload)+Ic(-Zline-Zload)
0=Ia+Ib+Ic

In matrix form

V=ZI

$$V := \begin{pmatrix} 208 \cdot e^{j \cdot 30deg} \\ 208 \cdot e^{j \cdot -90deg} \\ 0 \end{pmatrix} \qquad Z := \begin{pmatrix} 11 + j & -11 - j & 0 \\ 0 & 11 + j & -11 - j \\ 1 & 1 & 1 \end{pmatrix}$$

and I is the column vector of currents Ia, Ib, and Ic

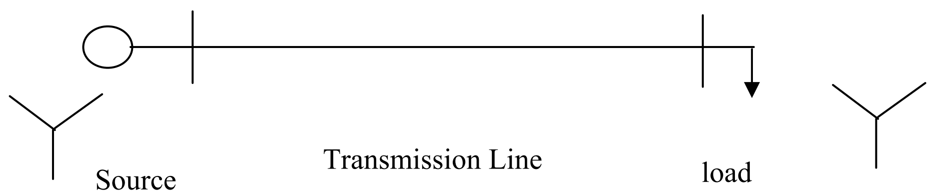
Then $I := Z^{-1} \cdot V$ $\quad I = \begin{pmatrix} 10.828 - 0.984i \\ -6.266 - 8.885i \\ -4.561 + 9.869i \end{pmatrix}$

ANALYSIS of BALANCED SYSTEMS USING PER PHASE OR LINE_NEUTRAL MODEL

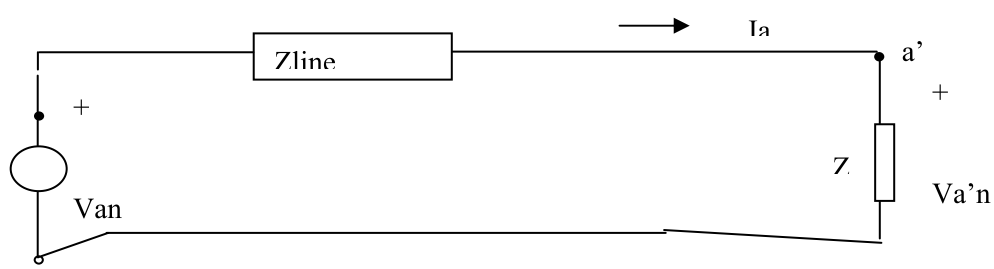
A balanced, positive sequence, 60 Hz, three-phase source supplies a balanced wye connected load through a three-phase transmission line. The neutral of the source is connected to that of the load through a wire with negligible impedance. The load impedance is 10 ohm/phase and line impedance is 1+j1 ohms per phase.

Find all line currents, line-neutral load voltages, line-line load voltages.

One line diagram



Per Phase Model or Line-neutral model



The model is valid whether or not the neutrals are tied, provided that the system is balanced

$Ia := \frac{Van}{Z + Zline}$ $\quad Ia = 10.828 - 0.984i \quad |Ia| = 10.872 \quad arg(Ia) = -5.194\,deg$

$Va'n := Van - Ia \cdot Zline \quad |Va'n| = 108.723 \quad arg(Va'n) = -5.194\,deg$

$Ib := Ia \cdot e^{j \cdot -120deg} \qquad Vb'n := Van \cdot e^{j \cdot -120deg}$

$Ic := Ib \cdot e^{j \cdot -120deg} \qquad Vc'n := Vb'n \cdot e^{j \cdot -120deg}$

$|Ib| = 10.872 \quad arg(Ib) = -125.194\,deg \quad |Vb'n| = 120.089 \quad arg(Vb'n) = -120\,deg$

$|Ic| = 10.872 \quad arg(Ic) = 114.806\,deg \quad |Vc'n| = 120.089 \quad arg(Vc'n) = 120\,deg$

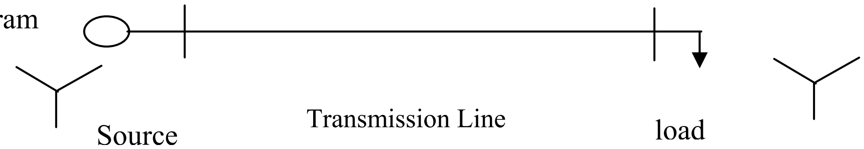
INSTANTANEOUS AND COMPLEX POWER

See Section 2.6

- The instantaneous power in each phase has oscillating components
- The total instantaneous power is constant
- The average power in each phase is REAL POWER per phase
    - $P1\varphi$ = Van Ia cos φ ; where $\varphi = /\underline{\theta_{Van}} - /\underline{\theta_{Ian}}$
- The REACTIVE POWER per phase is
    - $Q1\varphi$ = Van Ia sin φ ; where $\varphi = /\underline{\theta_{Van}} - /\underline{\theta_{Ian}}$
- The COMPLEX POWER per phase is
    - $S1\varphi$ = Van Ia* = $P1\varphi + j\, Q1\varphi$
- The total 3 phase complex power is defined as $S3\ \varphi = 3\ S\ 1\varphi$ = 3 Van Ia*
- Useful cook book formulas |S3
    - $P = \sqrt{3}$VLL IL cos φ where $\varphi = /\underline{\theta_{Van}} - /\underline{\theta_{Ian}}$, VLL is l-l voltage magnitude and IL is line current magnitude
    - $|S| = |S3\ \varphi| = \sqrt{3}$VLL IL

Continuing the example from the last lecture

One line diagram



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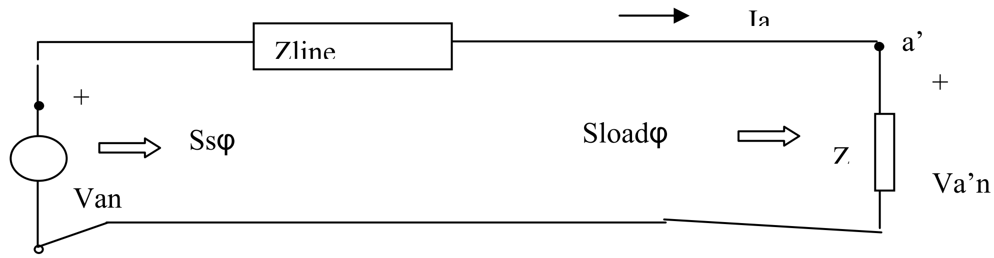


Remember, by convention, the given source voltage of 208 V is a LINE-LINE, RMS voltage

It is convenient to let Van be the voltage reference so let

Then $Van := \frac{208}{\sqrt{3}} \cdot e^{j \cdot 0deg}$ V

$Zline := 1 + j$ $Z := 10$ ohm

$Ia := \frac{Van}{Z + Zline}$ $Ia = 10.828 - 0.984i$ $|Ia| = 10.872$ $arg(Ia) = -5.194deg$ A

$Va'n := Van - Ia \cdot Zline$ $|Va'n| = 108.723$ $arg(Va'n) = -5.194deg$ V

Complex power

Delivered to Load $Sload\phi := Va'n \cdot \overline{Ia}$ $Sload\phi = 1.182 \times 10^3$ $\frac{VA}{phase}$

$Sload3\phi := 3 \cdot Va'n \cdot \overline{Ia}$ $Sload3\phi = 3.546 \times 10^3$ VA Total or Three-phase (Default)

Delivered by Source $Ss\phi := 3 \cdot Van \cdot \overline{Ia}$ $Ss\phi = 3.901 \times 10^3 + 354.623i$ $\frac{VA}{phase}$

$Ss3\phi := 3 \cdot Van \cdot \overline{Ia}$ $Ss3\phi = 3.901 \times 10^3 + 354.623i$ VA

Loss

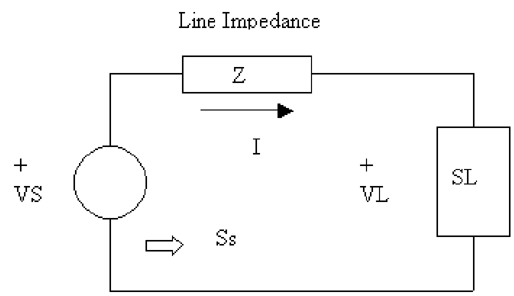
$Sloss3\phi := Ss3\phi - Sload3\phi$ $Sloss3\phi = 354.623 + 354.623i$ VA

From this point on we will drop the 3φ subscript on total 3 phase power.

PROBLEM SOLVING

Example 1 Three Phase Systems **Constant KVA load**
A three phase, 60 Hz, positive sequence source supplies a 39KVA, 0.9 pf lagging load through a transmission line of impedance 0.1+j0.1 ohm per phase. Determine the source voltage needed to obtain 208 V at the load. Also calculate the complex power supplied by the source.

Per phase or Line-Neutral model



Line Impedance $z := 0.1 + j\cdot 0.1$ ohm per phase

Load complex power $SL := 39000 e^{j\cdot acos(.9)}$ Total or three phase

$SL\phi := \frac{SL}{3}$ $SL\phi = 1.17\times 10^{4} + 5.667i\times 10^{3}$ VA per phase

Load Voltage $VLL := 208$ V l-l

$VL := \left(\frac{VLL}{\sqrt{3}}\right)\cdot e^{j\cdot 0}$ V line-neutral

So Line Current $IL := \overline{\left(\frac{SL\phi}{VL}\right)}$ $IL = 97.428 - 47.186i$ $|IL| = 108.253$ $arg(IL) = -25.842deg$

Then $VS := VL + z\cdot IL$ $|VS| = 134.644$ $arg(VS) = 2.138deg$

This answer is in line to neutral volts.

So magnitude of source vltage in line-line $134.644\sqrt{3} = 233.21$ V

Load Power $SL = 3.51\times 10^{4} + 1.7i\times 10^{4}$ VA Given, Three phase

Source Power $SS := 3\cdot VS\cdot \overline{IL}$ $SS = 3.862\times 10^{4} + 2.052i\times 10^{4}$ VA Three phase

Loss $Loss := SS - SL$ $Loss = 3.516\times 10^{3} + 3.516i\times 10^{3}$ VA

Can calculate Loss as follows

$Loss := (|IL|)^{2}\cdot z\cdot 3$ $Loss = 3.516\times 10^{3} + 3.516i\times 10^{3}$ VA

Source power factor $pfs := \cos(arg(VS) - arg(IL))$ $pfs = 0.883$ lag

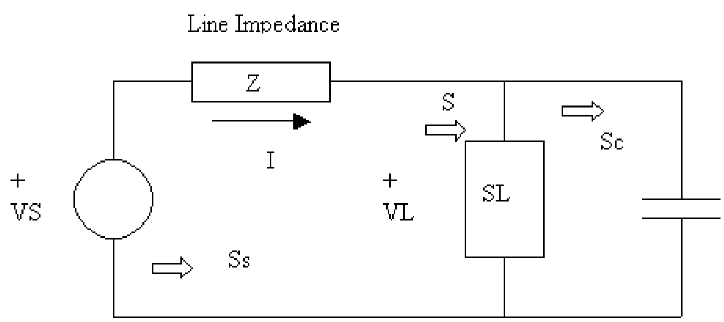
AN Aside

Cokbook approach to calculating load current

$IL := \left(\frac{|SL|}{\sqrt{3}\cdot VLL}\right)\cdot e^{i\cdot -acos(0.9)}$ $|IL| = 108.253$ $arg(IL) = -25.842deg$

Example 2: **Power Factor correction**

For the system in the previos example, correct the power factor to unity. Determine capacitor required, source voltage, and loss. Load voltage should be 208 V



Unless otherwise noted all powers are three phase, voltages are line-line, currents are line currents

Let Qc represent the reactive power SUPPLIED by the capacitor

Complex power delivered TO Capacitor is SC= - j QC

Let S represent load complex power after correction.

S = SL +SC = PL + j QL - jQC

Let ϕold represent the original power factor angle and ϕnew the desired one.

Then tan(ϕold) = QL/PL and tan(ϕnew)= (QL-QC)/PL

Note the real power must remain the same, while reactive power changes

Thus (QL-QC)/QL = tan(ϕnew)/tan(ϕold) so QC=QL-PLtan(ϕnew)

In our case the desired power factor is 1, so ϕnew=0. Solve for QC

$SL = 3.51\times 10^{4} + 1.7i\times 10^{4}$ VA $PL := Re(SL)$ $QL := Im(SL)$

$\phi new := 0$

$QC := QL - PL\cdot \tan(\phi new)$ $QC = 1.7\times 10^{4}$ VAR

After Correction

$SC := -j\cdot QC$ $S := SL + SC$ VA Three phase

$S\phi := \frac{S}{3}$ $S\phi = 1.17\times 10^{4}$ VA per phase

At load $VLL := 208$ (line − line)V

$VL := \frac{VLL}{\sqrt{3}}$ use this as phasor reference

So $IL := \overline{\left(\frac{S\phi}{VL}\right)}$ $IL = 97.428$ $|IL| = 97.428$ $\arg(IL) = 0\,deg$

Then $VS := VL + z\cdot IL$ $|VS| = 130.197$ $\arg(VS) = 4.292deg$

SOurce voltage needed $130.197\sqrt{3} = 225.508$ Vl − 1

Note improvement in voltage, current and loss

Load Power $SL = 3.51\times 10^{4} + 1.7i\times 10^{4}$

Source Power $SS := 3VS\cdot\overline{IL}$ $SS = 3.795\times 10^{4} + 2.848i\times 10^{3}$

Loss $Loss := SS - S$ $Loss = 2.848\times 10^{3} + 2.848i\times 10^{3}$

Example 3

a. Assume that the capacitor is wye connected; calculate the capacitance per phase
b. Assume that the capacitor is delta connected; calculate the capacitance per phase
Calculate the phasor line currents Ia, Ib, Ic
Calculate the phasor delta currents Iab, Ibc, Ica

So $IC := \overline{\left(\frac{\frac{SC}{3}}{\frac{VL}{\sqrt{3}}}\right)}$ $IC = 81.729i$

For wye

$ZC := \frac{VL}{IC}$ $ZC = -1.469i$ ohm

Then

$XC := -Im(ZC)$ $XC = 1.469$

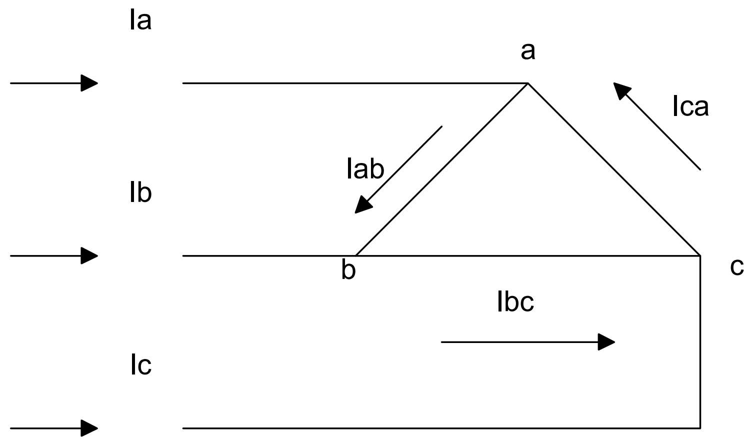
Since XC = 1/ ωC $C := \frac{1}{377\cdot XC}$ $C = 1.805\times 10^{-3}$

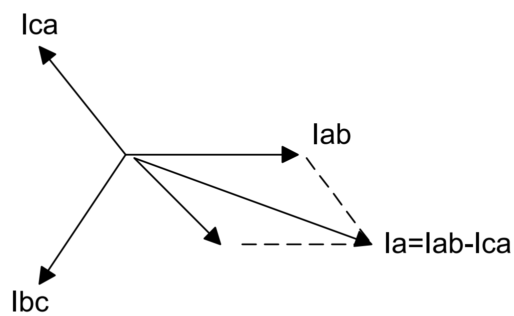
For delta $ZC := 3\cdot ZC$

$ZC = -4.408i$ ohm $XC := -Im(ZC)$

$XC = 4.408$ $C := \frac{1}{377\cdot XC}$ $C = 6.017\times 10^{-4}$

Note, compared to the wye connection I need less capacitance per phase but the capacitors must be designed for 208 rather than 120 V





$Ia := 6.287i$

$Iab := \left(\frac{1}{\sqrt{3}} \cdot Ia \cdot e^{i \cdot 30deg}\right)$

$Ib := Ia \cdot e^{i \cdot -120deg}$

$Ibc := \frac{1}{\sqrt{3}} \cdot Ib \cdot e^{i \cdot 30deg}$

Corrected

$Ic := Ia \cdot e^{i \cdot 120deg}$

$Ica := \frac{1}{\sqrt{3}} \cdot Ic \cdot e^{i \cdot 30deg}$

$|Iab| = 3.63$ $\arg(Iab) = 120deg$

$|Ibc| = 3.63$ $\arg(Ibc) = 7.01 \times 10^{-15} deg$

$|Ica| = 3.63$ $\arg(Ica) = -120deg$