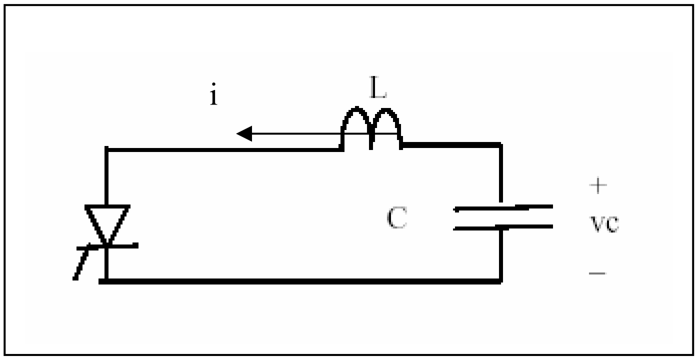


Assume SCR will turn on because it is forward biased

Since vc= Ldi/dt and I = C dvc/dt, the differential equation is

LCd^2vc/dt^2 +vc = 0 for t>0 and vc(0)=E   i(0) = Cdvc/dt$|_{t=0}$=0

The solution has the form vc(t) = K cos(ωt+θ) with ω= 1/√(LC)

Using the initial conditions

K cosθ = E and -Kω sin(θ) = 0 => θ=0 and K=E

Thus vc(t) = E cos(ωt)

And i(t) = Cdvc/dt = C Eω sin(ω t)

The current is positive for 0<t<ω/π so our initial assumption that the SCR is on is correct.

At ωt= π the current swigs negative so the SCR must turn off. The voltage vc(t) at this point is negative (-E), so the SCR will turn off. The current now remains zero and the capacitor remains charged at -E

We have Ri +Ldi/dt + vc = E and i = Cdvc/dt, yielding the differential equation

$LCd^2vc/dt^2 + R\,C\,dvc/dt + vc = E$ for t>0 vc(0)=0 and i(0) = 0

Use Laplace transforms

$Vc(s) = (E/s) / (LC\,s^2 + RC\,s + 1)$

Plug in numbers

$Vc(s) = (10 /s)/ (10^{-9}\,s^2 + 10^{-9}s+1)$

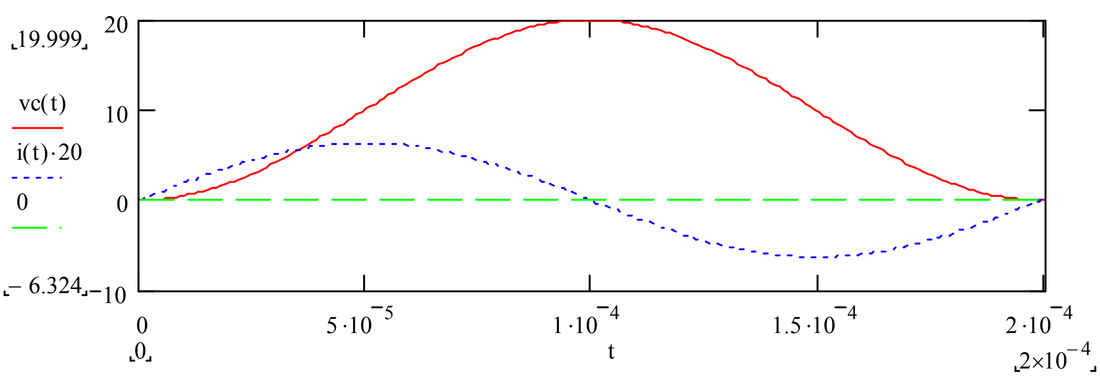
Inverting,

$$vc(t) := 10 - 10\cdot \exp\left(\frac{-1}{2}\cdot t\right)\cdot \cos\left(3.162\times 10^{4}\cdot t\right) - 2.5\times 10^{-9}\cdot \exp\left(\frac{-1}{2}\cdot t\right)\cdot\left(3.162\times 10^{4}\right)\cdot \sin\left(3.162\times 10^{4}\cdot t\right)$$

$$i(t) := 2.500410^{-6}\cdot \exp\left(\frac{-1}{2}\cdot t\right)\cdot \cos(31620\cdot t) + .3162\exp\left(\frac{-1}{2}\cdot t\right)\cdot \sin(31620\cdot t)$$

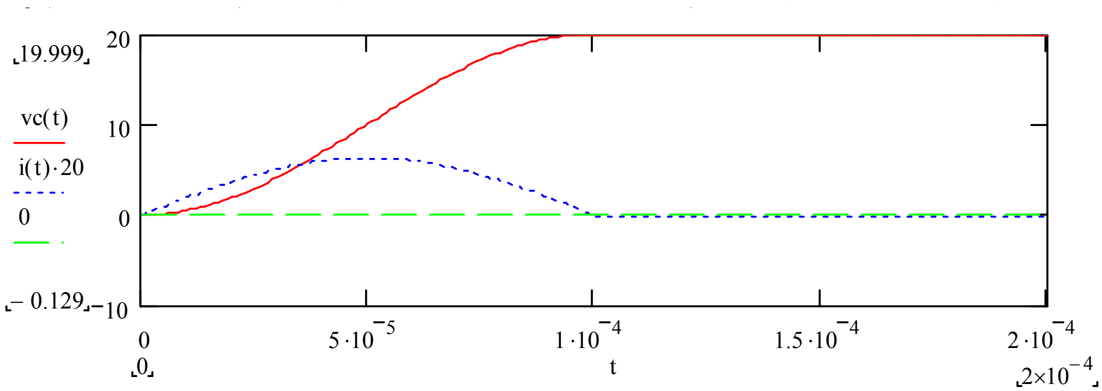
These responses are plotted below. The current has been multiplied by 20 for clarity.

The frequency of oscillation 31620 rad/s corresponds to ~5 kHz and a period of 200 uS

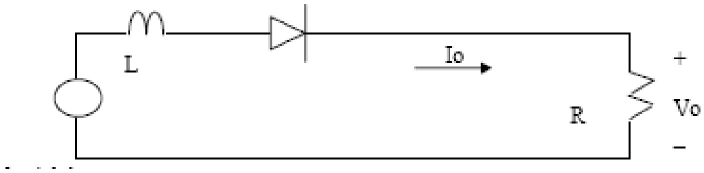


At ~100 us the current goes to zero and at this point vc=19.99 V> E. The diode will turn off.

The final solution is plotted below. The final voltage is < 2E



19.999
20
vc(t)
10
i(t)·20
0
0
−0.129
−10
0
$5 \cdot 10^{-5}$
$1 \cdot 10^{-4}$
$1.5 \cdot 10^{-4}$
$2 \cdot 10^{-4}$
0
t
2×10^{-4}



At t=0 the diode must turn on and the DE is

V=Ri+Ldi/dt   v=√2 V cos ωt   i(0)=0

The solution has the form

$i(t) = (\sqrt{2}\,V/|Z|)\cos(\omega t - \theta) + K e^{-tR/L}$

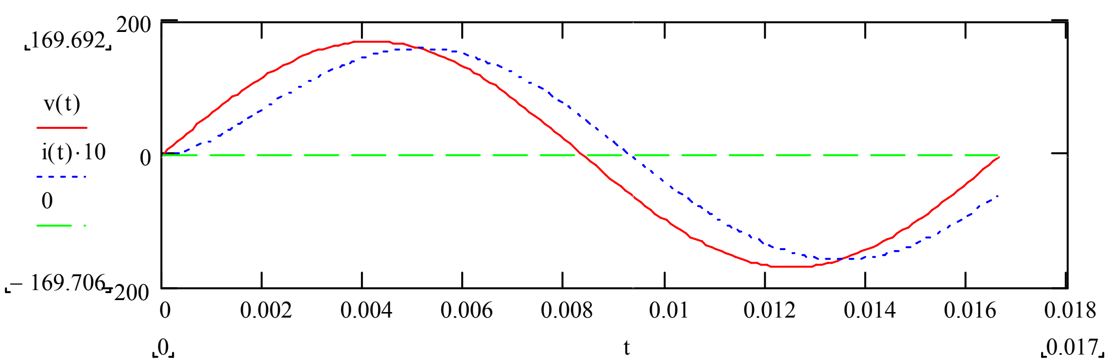
Using the initial condition

$0 = (\sqrt{2}\,V/|Z|)\cos(-\theta) + K$ so $K = -(\sqrt{2}\,V/|Z|)\cos(\theta)$

and,

$i(t) = (\sqrt{2}\,V/|Z|)[\cos(\omega t - \theta) - \cos(\theta)e^{-tR/L}]$

Using mathcad to plot i(t) we find i(t)=0 at t=9.4 mS

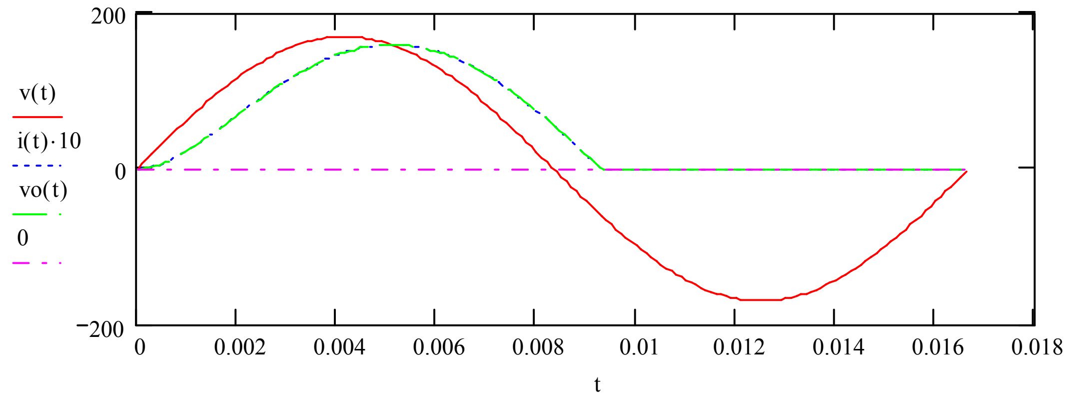


Therefore conduction ceases at 9.4 mS and resumes at the next positive half cycle

The complete solution is

$$i(t) := \begin{cases} \left(\sqrt{2}\cdot\frac{V}{Zmag}\right)\cdot\left(\sin(\omega\cdot t-\theta)+\sin(\theta)\cdot e^{\frac{-t\cdot R}{L}}\right) & \text{if } t < .0094 \\ 0 & \text{otherwise} \end{cases}$$

$$vo(t) := R\cdot i(t)$$



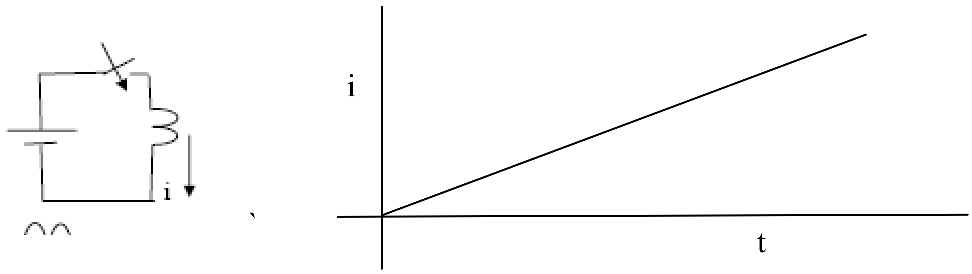
The average voltage is

frequency $f := 60$ Hz period $T := \frac{1}{60}$ $T = ▮$ mS

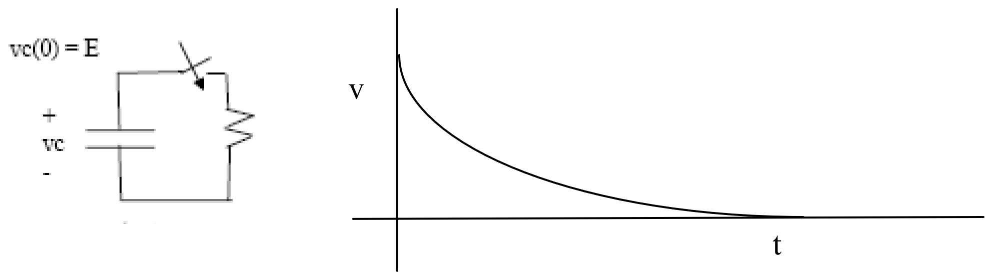
$$Vo := \left(\frac{1}{T}\right)\cdot\int_0^{0.0094} vo(t)\,dt \qquad Vo = ▮ \quad V$$

Recall from class that, absent inductance, the average voltage is $\sqrt{2}\cdot\frac{V}{\pi} = 54.019$ V

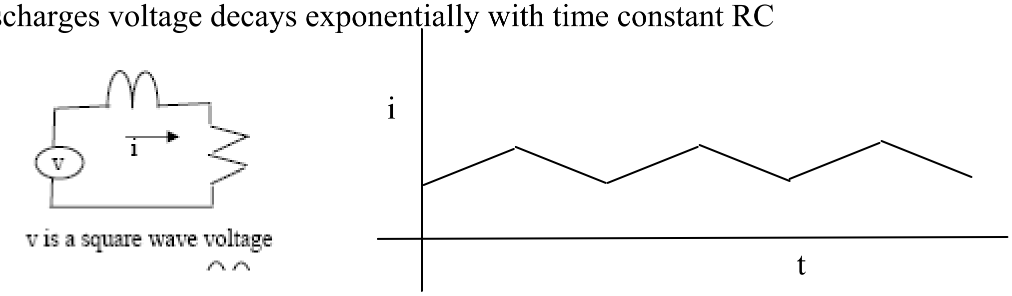
Thus source inductance leads to a voltage drop



Current builds up as a ramp since E= constant= Ldi/dt



Discharges voltage decays exponentially with time constant RC



Behaves as a low pass filter for current