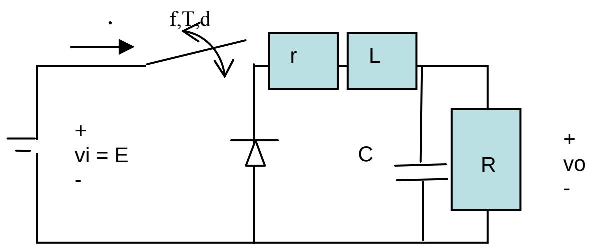
Part 1 A. Analysis and Simulation of Power Electronic Circuits (Continued)

Dynamics of the average- state-space averaging

Example 2 The buck converter

The circuit below is that of a step-down or buck dc-dc converter. The output is a dc voltage. Our goal is to derive the small signal dynamic model for average output voltage. Note that this time we have included the fact that the inductor does have some resistance, r.



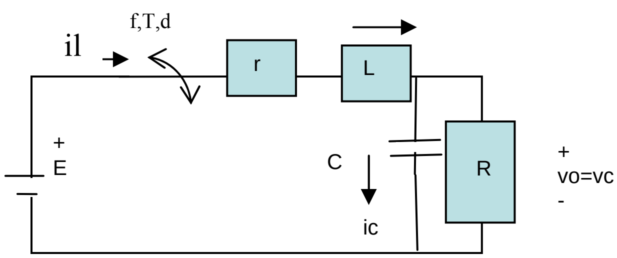
We will first work through circuit operation and develop the state space model for each circuit mode.

Step 1: Assuming CCM the circuit operates in two circuit modes in each switching period(K=2)

Step 2 Circuit Modes and state model

Mode 1 (k=1)

When the switch is on (Circuit mode 1) the diode is reverse- biased and off. Current builds up in the inductor. Assuming[1] that the capacitor voltage is less than E this inductor current supplies the load and charges the capacitor.



E = r il + L dil/dt + vc

il = ic + vc/R = C dvc/dt + vc/R

$$\frac{d}{dt}\begin{bmatrix} il \\ vc \end{bmatrix} = \begin{bmatrix} -r/L & -1/L \\ 1/C & -1/RC \end{bmatrix}\begin{bmatrix} il \\ vc \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} E$$

$$vc = [0 \quad 1]\begin{bmatrix} il \\ vc \end{bmatrix} + [0]E$$

[1] As usual we make reasonable assumptions and charge ahead. If answers turn out inconsistent with the assumption we will need to reconsider.

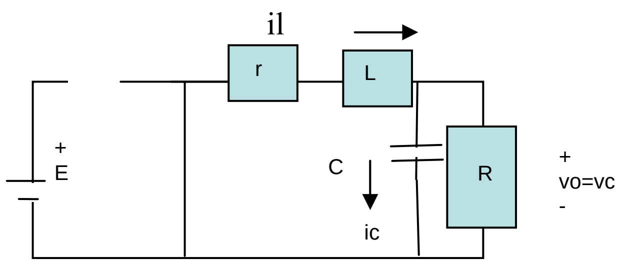
Thus,

$$A1 = \begin{bmatrix} -r/L & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad B1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \quad C1 = [0 \quad 1] \quad D1 = [0]$$

This mode lasts for time t=T1=DT

Mode 2 (k=2)

Then, when the switch is turned off (circuit mode 2), continuity of inductor current demands that the diode turn on. Current freewheels. Inductor current may initially charge the capacitor further but as, it decays and falls below resistor current the capacitor will begin discharging.



E = r il + L dil/dt + vc
il = ic + vc/R = C dvc/dt + vc/R

$$\frac{d}{dt}\begin{bmatrix} il \\ vc \end{bmatrix} = \begin{bmatrix} -r/L & -1/L \\ 1/C & -1/RC \end{bmatrix}\begin{bmatrix} il \\ vc \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}E$$

$$vc = [0 \quad 1]\begin{bmatrix} il \\ vc \end{bmatrix} + [0]E$$

Thus,

$$A1 = \begin{bmatrix} -r/L & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad B1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \quad C1 = [0 \quad 1] \quad D1 = [0]$$

This mode lasts for time t=T-T1=(1-D)T

Step 3:

Averaged Model.

$$\widetilde{A} = \frac{[T1A1 + (T - T1)A2]}{T} = \begin{bmatrix} -r/L & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad \widetilde{B} = [T1B1 + (T - T1)B2] = \begin{bmatrix} D/L \\ 0 \end{bmatrix}$$

$$\widetilde{C} = [T1C1 + (T - T1)C2] = [0 \quad 1] \qquad \widetilde{D} = [T1D1 + (T - T1)D2] = [0]$$

Step 4 Steady State Analysis

In the steady state $X = -\tilde{A}^{-1}\tilde{B}U$

$$\tilde{A}^{-1}\tilde{B} = \frac{-1}{(r/RLC + 1/LC)}\begin{bmatrix} -1/RC & 1/L \\ -1/C & -r/L \end{bmatrix}\begin{bmatrix} D/L \\ 0 \end{bmatrix} = \frac{1}{(r/RLC + 1/LC)}\begin{bmatrix} DE/RLC \\ DE/LC \end{bmatrix}$$

The output voltage is the second state variable, so its steady state or average value is

$$Vo = Vc = \frac{DE/LC}{(r/RLC + 1/LC)} = \frac{DE}{(r/R + 1)} = DE\frac{R}{R+r}$$

Ideally the output voltage is DE as we already know. The effect of inductor resistance is to create a voltage divider effect R/R+r, thereby reducing the average output voltage.

Step 5 Small signal dynamic model

The small signal model is

$$\tfrac{d}{dt}\Delta x = \hat{A}\Delta x + \hat{B}\Delta u \quad \Delta y = \hat{C}\Delta x + \hat{D}\Delta u$$

$$\tfrac{d}{dt}\begin{bmatrix} \Delta Il \\ \Delta Vc \end{bmatrix} = \begin{bmatrix} -r/L & -1/L \\ 1/C & -1/RC \end{bmatrix}\begin{bmatrix} \Delta Il \\ \Delta Vc \end{bmatrix} + \begin{bmatrix} E/L & 0 \\ 0 & D/L \end{bmatrix}\begin{bmatrix} \Delta D \\ \Delta E \end{bmatrix}$$

$$\Delta Vc = [0 \quad 1]\begin{bmatrix} \Delta Il \\ \Delta Vc \end{bmatrix} + [0]\begin{bmatrix} \Delta D \\ \Delta E \end{bmatrix}$$

Note now both the duty cycle change ΔD and source voltage change ΔE now appear as elements of the input vector U.
The transfer function form is

$$Y(s) = H(s)U(s) \qquad H(s) = \hat{C}(sI - \hat{A})^{-1}\hat{B} + \hat{D}$$

Noting that $$(sI - \hat{A}) = \begin{bmatrix} s + r/L & 1/L \\ -1/C & s + 1/RC \end{bmatrix}$$

and $$(sI - \hat{A})^{-1} = \frac{1}{(s + r/L)(s + 1/RC) + 1/LC}\begin{bmatrix} s + 1/RC & -1/L \\ 1/C & s + r/L \end{bmatrix}$$

$$H(s) = \frac{1}{(s + r/L)(s + 1/RC) + 1/LC}\begin{bmatrix} (s + 1/RC)E/L & -D/L^2 \\ E/LC & (s + r/L)D/L \end{bmatrix}$$

Suppose $\Delta E=0$. Then the transfer function $H_D(s)$ between duty ratio D and output Vc is

$$H_D(s) = \frac{E/LC}{(s + r/L)(s + 1/RC) + 1/LC}\Delta D(s)$$

$$H_D(s) = \frac{E}{LCs^2 + (rC + L/R)s + (1 + r/R)}\Delta D(s)$$

With this model we can design an appropriate feedback controller to regulate the output voltage.

Observations:

Step response:

In the previous analysis of the buck converter to create a 10 V , 10 A regulated supply from a 12 V dc battery, we used f= 5kHz, and D= 0.833. Given a 1 mH inductor we needed a 0.8 uF capacitor for 1% voltage ripple. The converter is well into CCM at D=0.833. Recall R= 10 ohm and let us assume r=0.1 ohm

The converter is operating in a steady state when we make a step change in duty of 0.1.

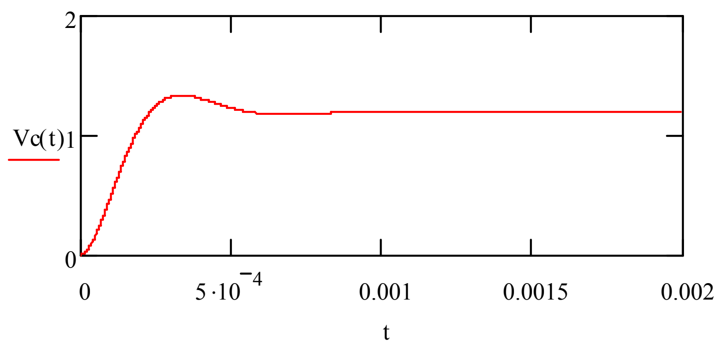
Thus $\Delta D(s)$= 0.1/s

$$Hd(s) := \frac{12}{.8e\text{-}8 \cdot s^2 + .1008e\text{-}3 s + 1.01}$$

$$Vc(s) := \frac{1.2}{\left(.8e\text{-}8 \cdot s^2 + .1008e\text{-}3 s + 1.01\right) \cdot s}$$

$$Vc(t) := 1.1881 - 1.1881 \cdot e^{(-6300.) \cdot t} \cdot \cos(9303.76t) - .8045 \cdot e^{(-6300.) \cdot t} \cdot \sin(9303.76t)$$

$$t := 0, .0000001 .. 0.002$$



For the chosen parameters the response is damped with a slight overshoot. The final change is ~1.2 V

Steady State Response

The dc gain (s=0) is $H_d(0)$= E/(1+r/R) =ER/(r+R). Thus for a steady state change ΔD the change in output is ΔD ER/(r+R). We notice the buck relationship ΔD E and the voltage divider effect R/R+r

The state space averaging approach is useful in deriving the dynamic equations. It also allows consideration of some non idealities.