Part 1 A. Analysis and Simulation of Power Electronic Circuits

The first part of the course focuses on understanding the basic concepts in power electronics through the analysis and simulation of power electronic circuits. Goals are to:

- Identify the most commonly used power electronic circuit forms or 'topologies' To use circuit analysis techniques understand the behavior of the circuit and to understand how and why the circuit performs its desired function. Design issues will be identified but not pursued in detail.
- To understand the control configurations needed to perform the desired function
- To review the calculation of performance measures such as average and rms voltage, power, and Fourier spectrum.
- To use simulation as an complement to analytical techniques

Power electronic topologies combine *switches, energy storage elements and switching patterns* to *obtain desired voltages and currents* at high power levels. Kassakian calls these "Form" and "Function", respectively. Although a very large variety of topologies are known they are all built from some very basic 'cells'. Standard techniques of circuit analysis will help us explore how the "form" achieves the "function"

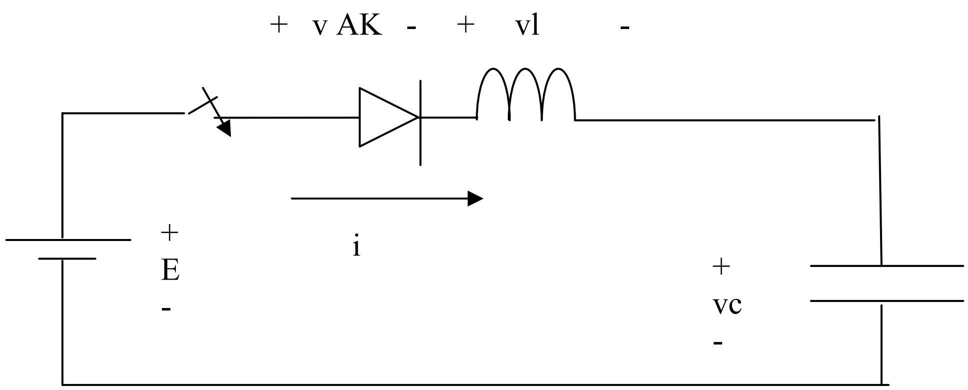
Power electronic circuits are inherently non linear since semiconductors are used as switches. Additional nonlinearities are offered by inductor saturation, for example. Ignoring the latter nonlinearities, the operation of the circuit can be developed in a step-by-step manner from a knowledge of the status of the switches. The basic steps can be summarized as follows:

1. Start with time t=0
2. Make reasonable assumptions regarding the status of each switch
3. Develop the differential equations for the resulting (linear) circuit
4. Develop circuit solution for $t \geq 0$
5. Verify that the assumed switch status in (a) is consistent with solution at t=0+; if not, start over.
6. Examine the solution to determine the time t1>t at which the assumed status is no longer correct

7. Modify switch status at time t= t1 and continue the process.

Once the solution is obtained we can calculate performance measures and relate these to design issues.

Example 1 A voltage doubler



With the switch S open there is initially no current in the inductor and the capacitor voltage is zero. The switch is closed at time t=0.

At time t=0 the diode appears to be forward biased so assume it will come on.

Assuming an ideal diode (zero forward voltage drop), the differential equation describing circuit response are developed as follows.

$E = L\, di(t)/dt + vc(t) \qquad i(t) = C\, dvc(t)/dt \quad i(0)=0 \quad vc(0)=0$

Thus,

$E = LC\, d^2vc(t)/dt^2 + vc(t) \qquad vc(0)=0 \quad dvc(t)/dt\,|_{t=0}=0$

We can use Laplace transforms to solve this second order equation

$E/s = LCs^2\, Vc(s) - s\, vc(0) - dvc(t)/dt\,|_{t=0} + sVc(s)$

Thus,

$Vc(s) = E / s\,(1+LCs^2)$

Inverting,

$vc(t) = E\,(1-\cos(\omega t)) \qquad \omega = 1/\sqrt{(LC)}$

Then,

i(t)= (E/Zc) sin(ωt)   and   vl(t)= E cos(ωt)

This solution holds until such time as a switch changes state. The solution is illustrated next.

Now at , the current i(t) swings negative and the diode must turn off.

Assuming that the diode is off for ωt > π, we have

i(t) = 0   vc(t)=E

$E := 10$ V $L := 0.001$ H $C := 0.000001$ F

The diode remains reverse biased (vAK=-E) and there is no further change.

$\omega := \frac{1}{\sqrt{L \cdot C}}$ $\omega = 3.162 \times 10^{4}$ $\frac{rad}{s}$ $f := \frac{\omega}{2 \cdot \pi}$ $f = 5.033 \times 10^{3}$ Hz

$vc(t) := E \cdot (1 - \cos(\omega \cdot t))$

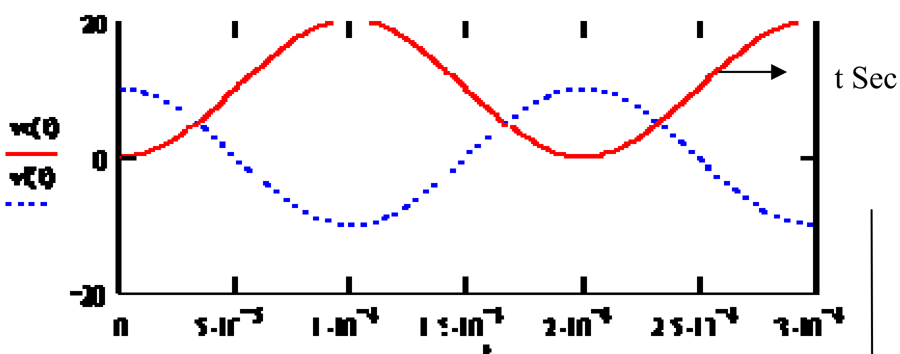
The final solution is shown below

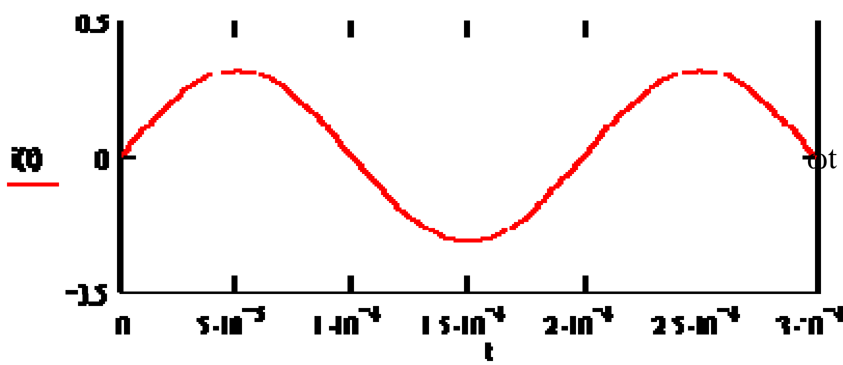
$Zc := \sqrt{\frac{L}{C}}$ $Zc = 31.623$ ohm

$i(t) := \left(\frac{E}{Zc}\right) \cdot \sin(\omega \cdot t)$

$vl(t) := E \cdot \cos(\omega \cdot t)$

$t := 0, .000002 .. 0.0003$



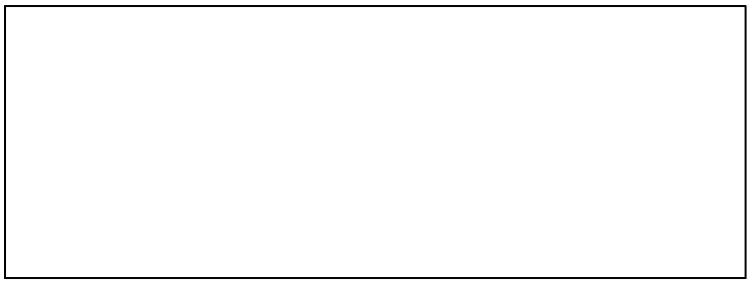


ωt = π/2   ωt = π

$$vc(t) := \begin{cases} E\cdot(1 - \cos(\omega \cdot t)) & \text{if } \omega \cdot t \le \pi \\ 2E & \text{otherwise} \end{cases}$$

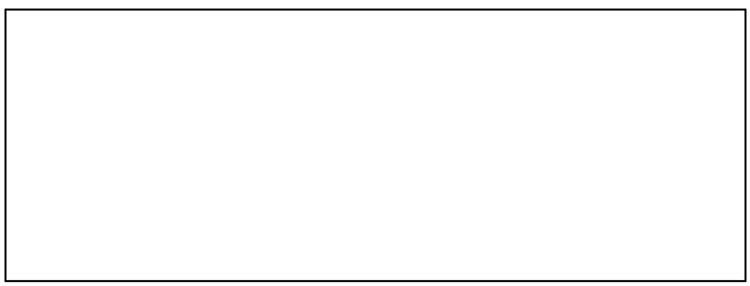
$$i(t) := \begin{cases} \left(\frac{E}{Zc}\right)\cdot \sin(\omega \cdot t) & \text{if } \omega \cdot t \le \pi \\ 0 & \text{otherwise} \end{cases}$$

vc(t)



t

i(t)



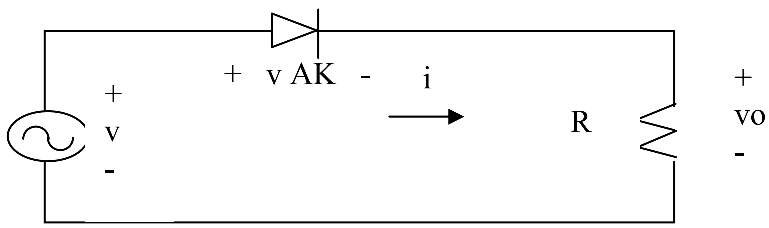
t

The solution appears contraintuitive – the capacitor voltage doubles; also, at ωt=π/2 the capacitor voltage rises above source voltage and the diode appears reverse biased! At ωt=π/2, if the diode were to turn off the inductor current would have to drop top zero instantaneously. However notice that the inductor voltage swings negative, keeping the diode forward biased and on. The diode turns off only when the current naturally goes to zero in the resonant cycle. Another way of looking at things is that the energy supplied(stored) in the inductor in the first quarter cycle must be removed as the current decay. Because of the diode, this energy can only go into the capacitor, thus doubling the voltage.

The diode LC "cell", and the responses derived for this simple circuit, are both basic to the operation of many power electronic converters.

Example 2 Single-phase half-wave rectifier

With this well-known circuit we will continue to develop circuit analysis principles and will also review the calculation of quantities such as average and rms voltage and power.



Starting with the positive cycle (t=0+), the diode appears forward biased. Assume it comes on. Then,

vo(t)=vi(t)   io(t) = vo(t)/R >0

These are consistent with the assumption that the diode is on.

AT the end of the positive half cycle the current is zero and the diode strats to see a reverse bias. Thus the diode turns off

vo(t)=0   io(t) = 0

Again, this is consistent our assumption.

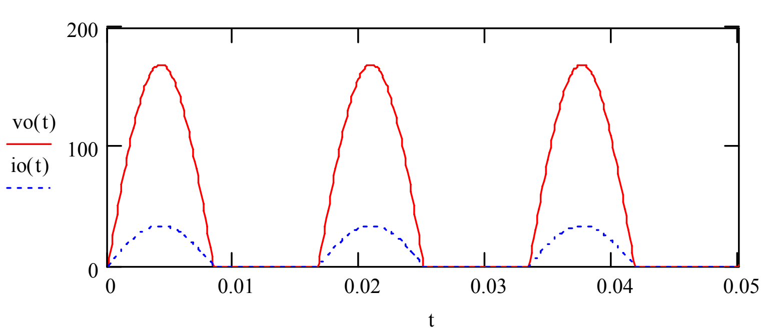
From here on the pattern repeats each positive and negative cycle, resulting in the waveforms shown below.

$\omega := 377 \ \frac{rad}{s}$ $R := 5$ ohm

$v(t) := 169 \cdot \sin(\omega \cdot t)$ correspond to 120 V ac rms

$vo(t) := \begin{cases} v(t) & \text{if } 0 \le \mathrm{mod}(\omega \cdot t, 2 \cdot \pi) < \pi \\ 0 & \text{otherwise} \end{cases}$ $io(t) := \frac{vo(t)}{R}$

$t := 0, .0001 .. .05$ In plotting chose a small enough step, 0.1 mS is good when ac period is 16mS



We will often need to calculate average and rms values of voltage and current and the real power delivered to the load.

For periodic waveforms f(θ)=f(ωt), recall:

Average or 'dc' value $F = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta$

RMS $F_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} f^2(\theta)d\theta}$

Real Power If the voltage across the load is v(t) and the current through the load is i(t)

$$P = \frac{1}{T}\int_0^{2\pi} v(t)i(t)dt = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)i(\theta)d\theta$$

In our example let v(t)= Vm sin(ωt) = Vm sin(θ)

Then,

$$V = \frac{1}{2\pi}\int_0^{\pi} Vm\sin(\theta)d\theta = Vm/\pi$$

Alternatively,

$$V = \frac{1}{T}\int_0^{T/2} Vm\sin(\omega t)dt = Vm/\pi$$

Similarly,

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} Vm^2\sin^2(\theta)d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} Vm^2\,\frac{(1-\cos(2\theta))}{2}\,d\theta} = Vm/2$$

and,

$$V_{rms} = \frac{1}{2\pi}\int_0^{\pi} v(\theta)i(\theta)d\theta = \frac{1}{2\pi}\int_0^{\pi}\frac{Vm^2\sin^2(\theta)}{R}d\theta == \frac{Vm^2}{4R}$$

Note P = $Vrms^2$ /R; P is not V^2/R unles v(t) is purely dc.

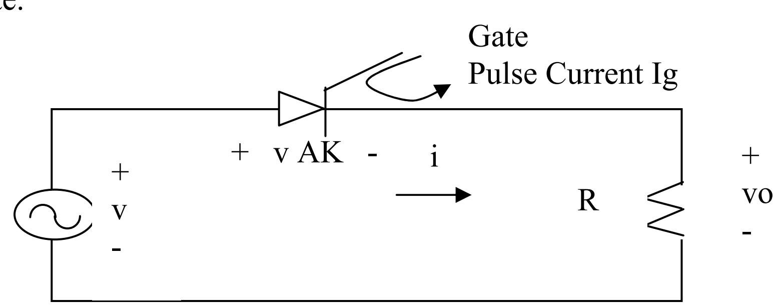
Example 3 Single-phase, half-wave, controlled rectifier

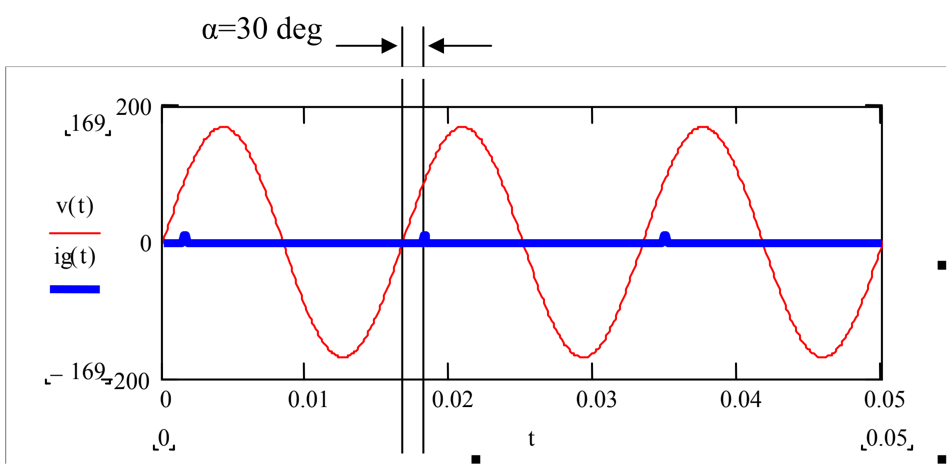
The rectifier in the previous example produces an output voltage with a dc component plus time varying components. The level of the dc component is directly proportion to the peak of the ac voltage and cannot be changed.

Replacing the diode by an Silicon Controlled Rectifier (SCR) allows us to control the average(dc) output voltage.

A control circuit, the 'firing circuit' is required and operates as follows. The controller detects each, positive going, zero crossing of the ac voltage. After a time delay Td secs, or $\alpha = \omega t$ degrees on the sine wave, the controller supplies a positive pulse of current to the gate-cathode circuit of the SCR.

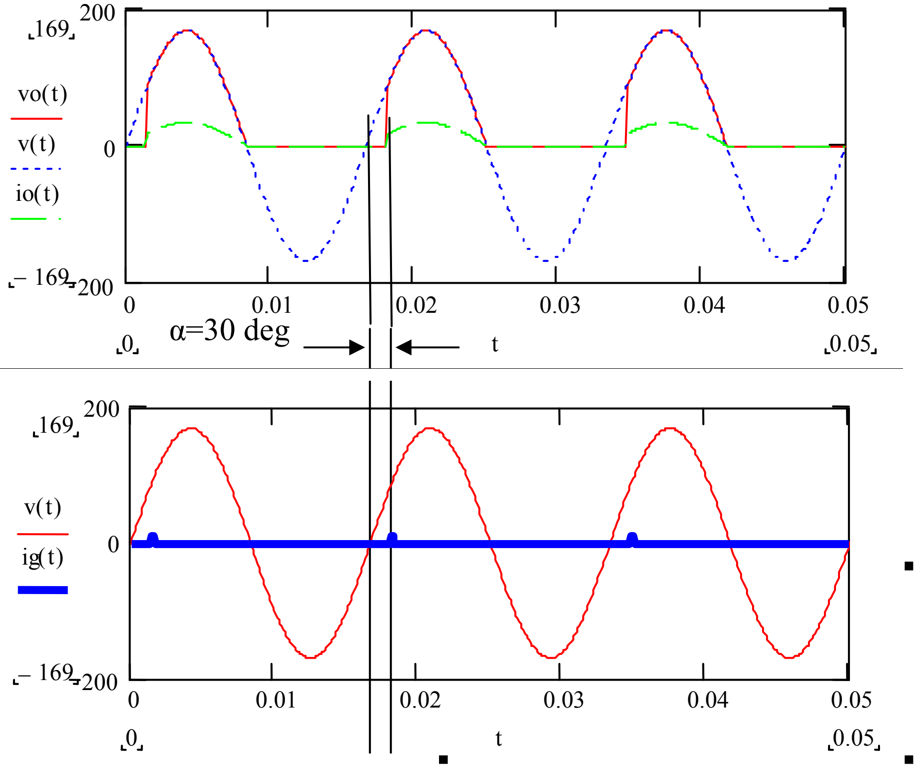
Recall that if an SCR is forward-biased, and a positive gate current pulse is applied, and if a positive current can flow from anode to cathode, the SCR will latch on. If the current is reduced to zero and a reverse bias is applied the SCR will turn off. The SCR will not turn on even if forward bias unless it is already on, or if a positive current pulse is applied to the gate.





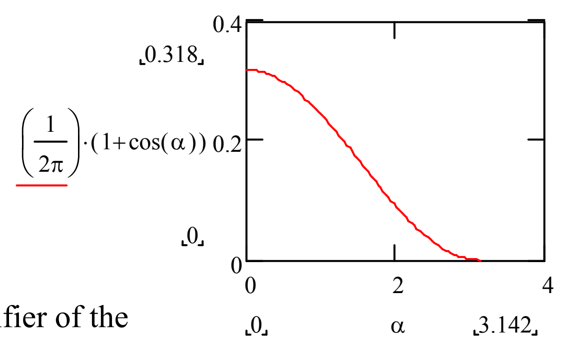
Thus the SCR in this circuit will turn on when the gate pulse appears and will turn off when the current naturally goes to zero and source voltage goes negative at ωt=π.

The resulting waveforms are shown below.



The average or dc output voltage is

$$V = \frac{1}{2\pi}\int_{\alpha}^{\pi} Vm\sin(\theta)d\theta = Vm(1+\cos\alpha)/2\pi$$



For α=0 the circuit behaves as the diode rectifier of the previous example. As α increases the dc voltage out put decrease. For α = π the output voltage is zero