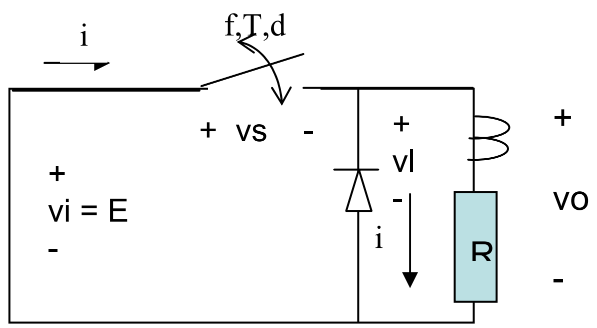
Part 1 A. Analysis and Simulation of Power Electronic Circuits (Continued)

Detailed, time-domain solution for periodic switching

Example 1: Chopper with inductive filter and free wheeling diode

We will now be interested in an exact solution for current in the practical chopper circuit below.

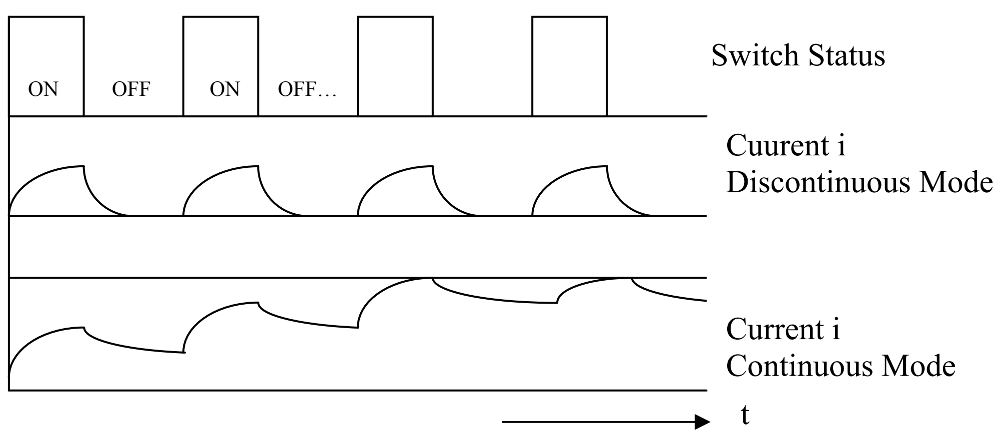


The switch is operated under pulse width modulation at switching frequency f, period T=1/f, and duty ratio D.

Qualitative picture:

When the switch is on the diode is reverse biased and off. Current builds up in the inductor. When the switch is turned off the inductor forward-biases the diode since its current cannot change instantaneously. Current freewheels and decays. Two cases are possible:

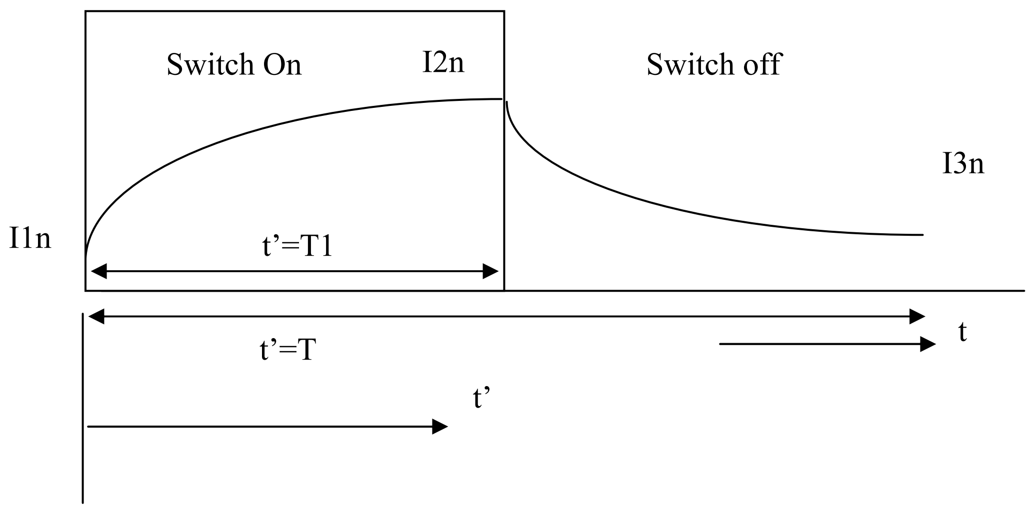
a. If the current decays to zero when the switch is off, then the next cycle starts afresh from zero current. This is the discontinuous conduction mode.
b. If the current does not decay to zero, then the current increase further in the next cycle, decays and so on, until a steady state is reached. The current never falls to zero in this continuous conduction mode.



Analysis:

We will assume the continuous conduction mode(CCM) and focus on one cycle.

Let	n	denote the cycle number
	I1n	denote the current at the beginning of cycle n
	I2n	denote the current when the switch turns off in cycle n
	I3n	denote the current at the end of cycle n
	t	denote absolute time
	t'	denote time measured from the start of cycle n
Note	t' = t-nT	



For time $0<t'<T1$, i.e., $nT<t<nT+T1$ The switch is on and the diode is off

The circuit equation is

$E = Ri + L\, di/dt'$ with $i(t'=0) = I1_n$

Solving the differential equation

$$i(t') = I1_n\, e^{(-t' R/L)} + (E/R)(1- e^{(-t' R/L)})$$

The switch turns off at time $t'=T1$ and at this time

$$I2_n = i(t'=T1) = I1_n\, e^{(-T1\, R/L)} + (E/R)(1- e^{(-T1\, R/L)})$$

Next for time $T1<t'<T$ the switch turns off. Since current is not zero the diode must come on.

The circuit equation is

$0 = Ri + L\, di/dt'$ with $i(t'=T1) = I2_n$

Solving the differential equation

$i(t') = I2_n\, e^{[-(t'-T1)\ R/L]}$

Thus at time $t' = T$

$I3_n = i(t'=T) = I2_n\, e^{[-(T-T1)\ R/L]}$

Summarizing, for cycle n, given the initial current $I1_n$ we get

$i(t) = I1_n\, e^{[-(t-nT)\ R/L]} + (E/R)(1- e^{[-(t-nT)\ R/L)}]$ for $nT<t<nT+T1$

$i(t) = I2_n\, e^{[-(t-nT)\ R/L]} + (E/R)(1- e^{[-(t-nT)\ R/L)}]$ for $nT+T1<t<(n+1)T$

where,

$I2_n = I1_n\, e^{(-T1\ R/L)} + (E/R)(1- e^{(-T1\ R/L)})$

$I1_{n+1} = I3_n = I2_n\, e^{[-(T-T1)\ R/L]}$

Notice that the current at the end of cycle n, $I3_n$, equals the current, $I1_{n+1}$, at the beginning of the next cycle.

Using these equations for successive cycles starting with n=0 we can plot the complete response as shown in the mathcad implementation that follows.

Let $R := 10$ $L := 0.03$ $E := 100$

Switcing frequency $f := 1000$ Hz Duty $D := 0.4$

Period $T := \frac{1}{f}$ Swirch on time $T1 := D \cdot T$

$T = 1 \times 10^{-3}$ s $T1 = 4 \times 10^{-4}$ s

Initialcurrent at the start ofcycle 0

Calculate I1 and I2 for succesive values of n

$n := 1, 2 .. 50$ $I1_n := 0$ $I2_n := 0$

$n := 0, 1 .. 50$

$$\begin{pmatrix} I2_n \\ I1_{n+1} \end{pmatrix} := \begin{bmatrix} I1_n \cdot e^{\frac{-T1 \cdot R}{L}} + \left(\frac{E}{R}\right)\left(1 - e^{\frac{-T1 \cdot R}{L}}\right) \\ \left[I1_n \cdot e^{\frac{-(T1 \cdot R)}{L}} + \left(\frac{E}{R}\right) \cdot \left(1 - e^{\frac{-T1 \cdot R}{L}}\right)\right] e^{-R \cdot \frac{(T - T1)}{L}} \end{bmatrix}$$

Now we can write the expressions for current i(t)

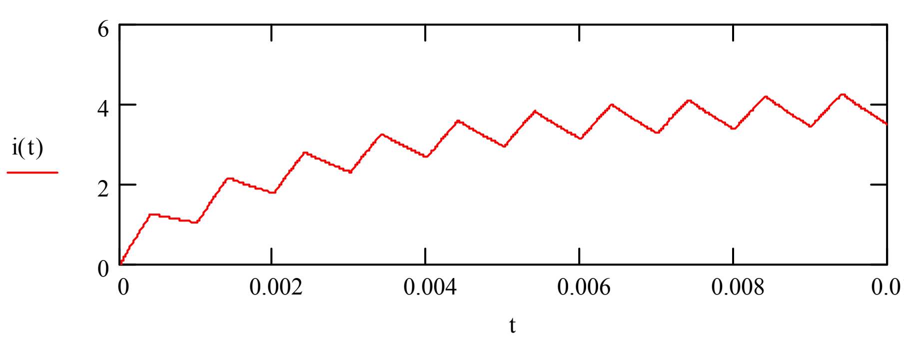
First define formulas for cycle # n and relative time t' as a function of time t

$n(t) := \mathrm{floor}\left(\frac{t}{T}\right)$ $t'(t) := t - n(t) \cdot T$

Now define i(t)

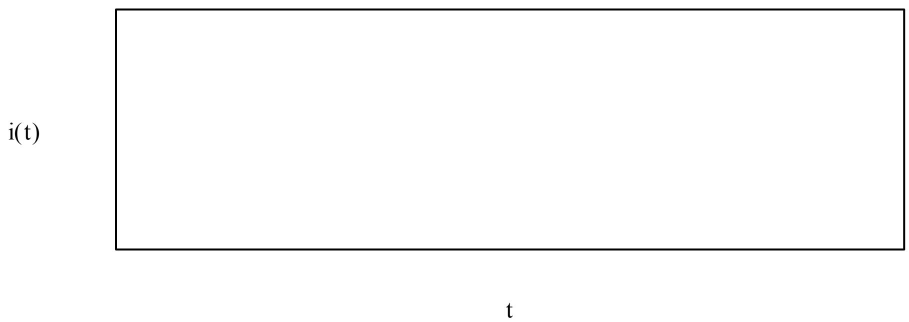
$$i(t) := \begin{cases} I1_{n(t)} \cdot e^{\frac{-t'(t) \cdot R}{L}} + \left(\frac{E}{R}\right) \cdot \left(1 - e^{\frac{-t'(t) \cdot R}{L}}\right) & \text{if } t'(t) < T1 \\ I2_{n(t)} \cdot e^{\left[\frac{-[(t'(t) - T1) \cdot R]}{L}\right]} & \text{otherwise} \end{cases}$$

$t := 0, .00001 .. .01$



If we lower the inductance L we get the discontinuous conduction mode

R := 10 L := 0.0005 E := 10(



Analysis of Steady state

Example 2: Chopper with inductive filter and free wheeling diode

Notice that in both cases of the chopper that a periodic steady state establishes in which the output current i(t) has a dc component with superimposed ripple. In CCM steady state is achieve in ~ 10-15 mS (The time constant τ= L/R is 3 ms). The ripple is small relative to the dc value.

In DCM the ripple is large and, technically, steady state is established in the first cycle.

We will always be interested in calculating the average value and the ripple.

In the periodic steady state

$I1_{n+1} = I1_n \qquad I1_{n+1} = I1_n$

Denote these steady state values by $\tilde{I}1$ and $\tilde{I}2$, respectively.
Since,

$$\tilde{I}1 = \tilde{I}2\ e^{[-(T-T1)\ R/L]}$$

and,

$$\tilde{I}2 = \tilde{I}1\ e^{(-T1\ R/L)} + (E/R)(1- e^{(-T1\ R/L)})$$

$\tilde{I}1 = [\tilde{I}1\, e^{(-T1\,R/L)} + (E/R)(1-e^{(-T1\,R/L)})]\, e^{[-(T-T1)\,R/L]}$

Thus,

$\tilde{I}1 = [\tilde{I}1\, e^{(-T1\,R/L)} + (E/R)(1-e^{(-T1\,R/L)})]\, e^{[-(T-T1)\,R/L]}$

$\tilde{I}1 = \tilde{I}1\, e^{(-T1\,R/L)}\, e^{[-(T-T1)\,R/L]} + (E/R)(1-e^{(-T1\,R/L)})]\, e^{[-(T-T1)\,R/L]}$

$\tilde{I}1\,(1-\; e^{(-T1\,R/L)}\, e^{[-(T-T1)\,R/L]}) = (E/R)(1-e^{(-T1\,R/L)})]\, e^{[-(T-T1)\,R/L]}$

So,

$\tilde{I}1 = ((E/R)(1-e^{(-T1\,R/L)})]\, e^{[-(T-T1)\,R/L]}) \,/\, (1-\; e^{(-T\,R/L)})$

And,

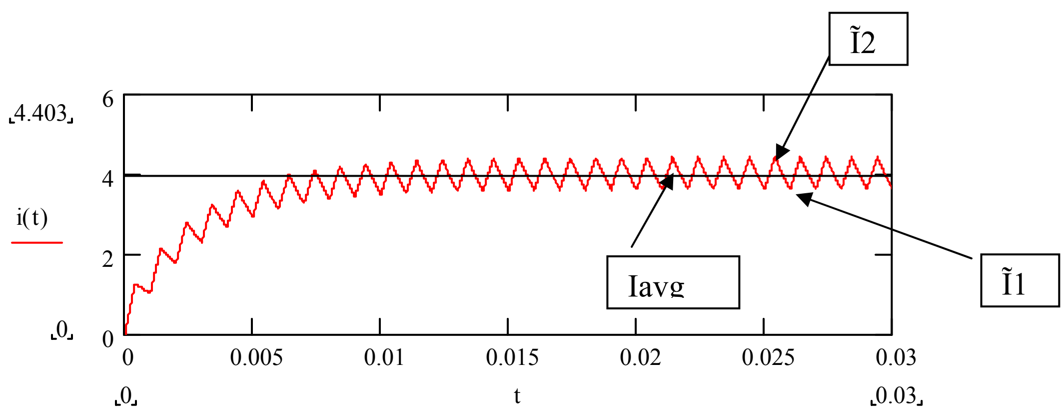
$\tilde{I}2 = ((E/R)(1-e^{(-T1\,R/L)})]\, e^{[-(T-T1)\,R/L]}) \,/[\,(1-\; e^{(-T\,R/L)})\, e^{[-(T-T1)\,R/L]}]$

Referring to Example 1 with R= 10 ohm, L= 0.03 H and E=100 V, T=1mS and T1=0.4 mS

$\tilde{I}1 = 3.605$ A

$\tilde{I}2 = 4.404$ A

These values can be seen at the tail end of the plot for i(t) below



The average value Iavg is,

$$Iavg = \frac{1}{T}\left[\int_{0}^{T1}\{\tilde{I}1e^{-tR/L} + (E/R)(1-e^{-tR/L})\}dt + \int_{T1}^{T}\{\tilde{I}2e^{-(t-T1)R/L}\}dt\right]$$

Performing the calculation we find Iavg = 4A

For the DCM we get

$\tilde{I}1 = 0$ A

$\tilde{I}2 = 10$ A

Iavg=3.5 A

In simpler converters, with idealized component models, it is possible to develop detailed analytical solutions of response. The exercise let's us understand basic waveforms in converter circuits. In general, periodic switching results in an initial transient that settles to a periodic steady state characterized by the desired output (a dc current in the chopper) along with unwanted components (ripple in the chopper). The response provides complete insight into the converter needed for design.

It is rarely possible to develop analytical solutions in non-ideal cases. This is where simulation comes in.