Part 1 A. Analysis and Simulation of Power Electronic Circuits (Continued)

Averaging Principles for Power electronic Circuit analysis

In the previous lecture we developed a complete analytical solution for periodically switched circuits, specifically, the chopper circuit. Such solutions are at best difficult to obtain. However, many features of the solution can be very effectively derived using the so-called averaging principle. The principle itself is exact, but the way we apply it can involve some idealization and approximation.

The most important application of the averaging principle is in deriving formulas for average voltages and currents and deriving models for the dynamics of average values in terms of input voltages, duty cycles, modulation strategies and circuit parameters.

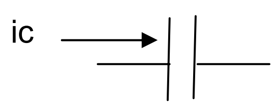
The principle:

We have now seen that the majority o power electronic circuits comprise of periodically switched devices with inductive and capacitive elements.

Consider a capacitor in such circuit: If there is an average dc current through the capacitor, it would continually charge and the voltage will keep building up. This is inconsistent with a periodic steady state.

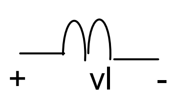
Therefore, in a periodic steady state condition the average current in each capacitor in the circuit must be zero. If T represents the switching frequency, and ic(t) the current through a capacitor

$$\frac{1}{T}\int_0^T ic(t)dt = 0$$



Similarly, in a periodic steady state condition the average voltage across each inductor in the circuit must be zero. Denote inductor voltage by vl(t),

$$\frac{1}{T}\int_0^T vl(t)dt = 0$$



Application of the averaging principle:

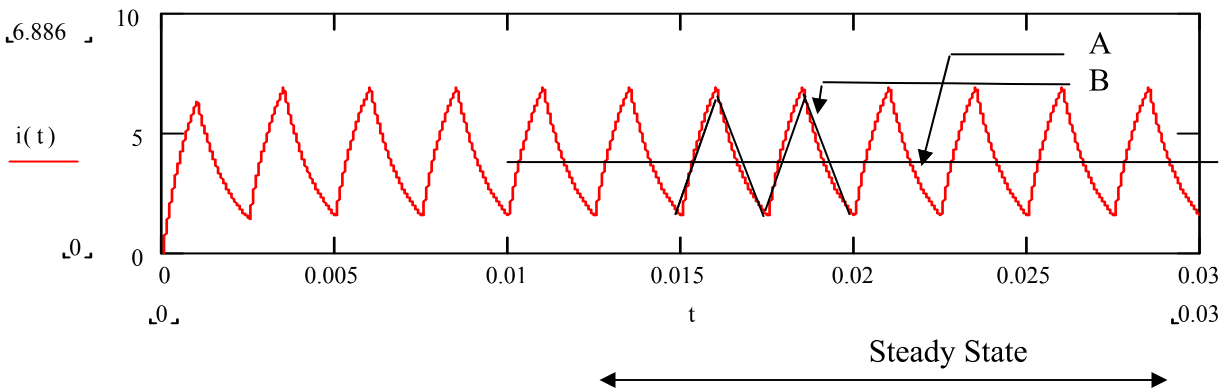
In applying the principle we begin by examining the operating modes of the circuit and sketching approximate but called waveforms of circuit quantities such as capacitor currents and inductor voltages.

Next we make appropriate approximations to the waveforms to make the integration involved in the averaging principle easier.

Finally, useful analytical formulas can be drive by solving the integral and simplifying.

Typical Approximation:

Recall the output current waveform, shown below, for the chopper circuit.

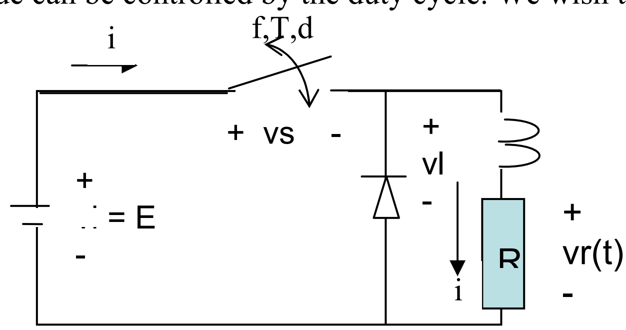


Some common approximations are described below:

A. The zeroth order approximation would replace the current in the steady state region by its average Iavg
B. The first order approximation would model the exponential rise and fall as a triangle wave

Example 1: Chopper with inductive filter and free wheeling diode

Recall that the chopper below converts the dc voltage into a dc current whose dc or average value can be controlled by the duty cycle. We wish to determine the average load current



We first develop the approximate waveform of current i(t) and inductor voltage vl(t) in the steady state assuming CCM

The switch is operated under pulse width modulation at switching frequency f, period T=1/f, and duty ratio D.

When the switch is on the diode is reverse biased and off. Current builds up in the inductor. The inductor voltage is

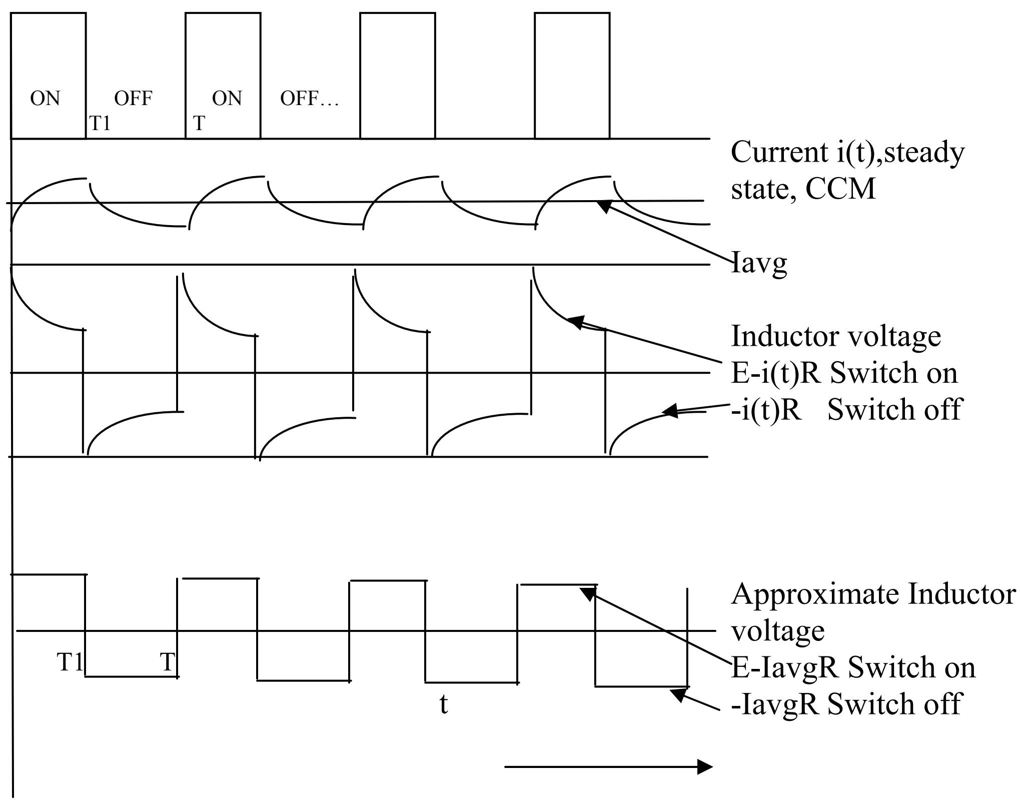
vl(t)= L di(t)/t = E-vr(t)= E- i(t) R

When the switch is turned off the inductor forward-biases the diode since its current cannot change instantaneously. Current freewheels and decays. The inductor voltage is

vl(t)= L di(t)/t = -vr(t)= - i(t) R

The switch status at waveforms f steady state inductor current and voltage are shown below. If we ignore ripple in the output current I and assume it to be constant at its average value, Iavg, then the inductor voltage becomes a rectangular ac wave given by

vl(t)= E- Iavg(t) R if the switch is on
vl(t)= - Iavg(t) R if he switch is off



Now we can derive a formula for Iavg by using the averaging principle for inductors. Integrate the voltage across the inductor for one cycle and setting this integral to zero gives

(E-Iavg R) T1 + (-Iavg R)(T-T1) = 0

Solving for Iavg,

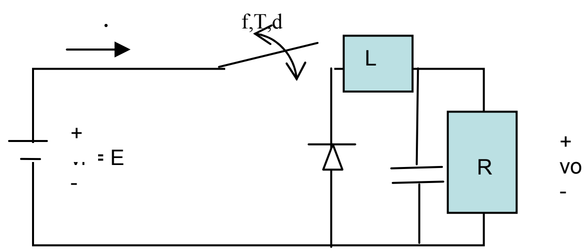
Iavg = (E/R) (T1/T) = D E/ R

Thus we find that the average current is directly proportional to D

Notice that using the average current as an approximation for i(t) greatly simplifies the integration step. Also keep in mind that we have assuming ideal devices, and ignored resistance

Example 2 The buck converter

The circuit below is that of a step-down or buck dc-dc converter. The output is a dc voltage. Our goal is to derive a formula for the average output dc voltage



Step 1

We will first work through circuit operation and sketch waveforms.

When the switch is on (Circuit mode 1) the diode is reverse- biased and off. Current builds up in the inductor. Assuming[1] that the capacitor voltage is less than E this inductor current supplies the load and charges the capacitor. Then, when the switch is turned off (circuit mode 2), continuity of inductor current demands that the diode turn on. Current freewheels. Inductor current may initially charge the capacitor further but as, it decays and falls below resistor current the capacitor will begin discharging. Again, both CCM and DCM are possible. We will assume CCM.

The circuits for each circuit mode and the approximate voltage and current waveforms are shown on the next page.

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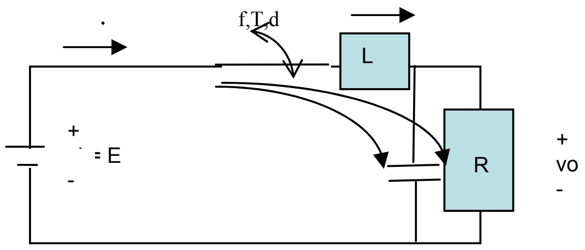
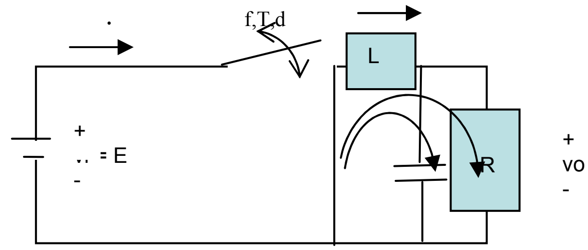
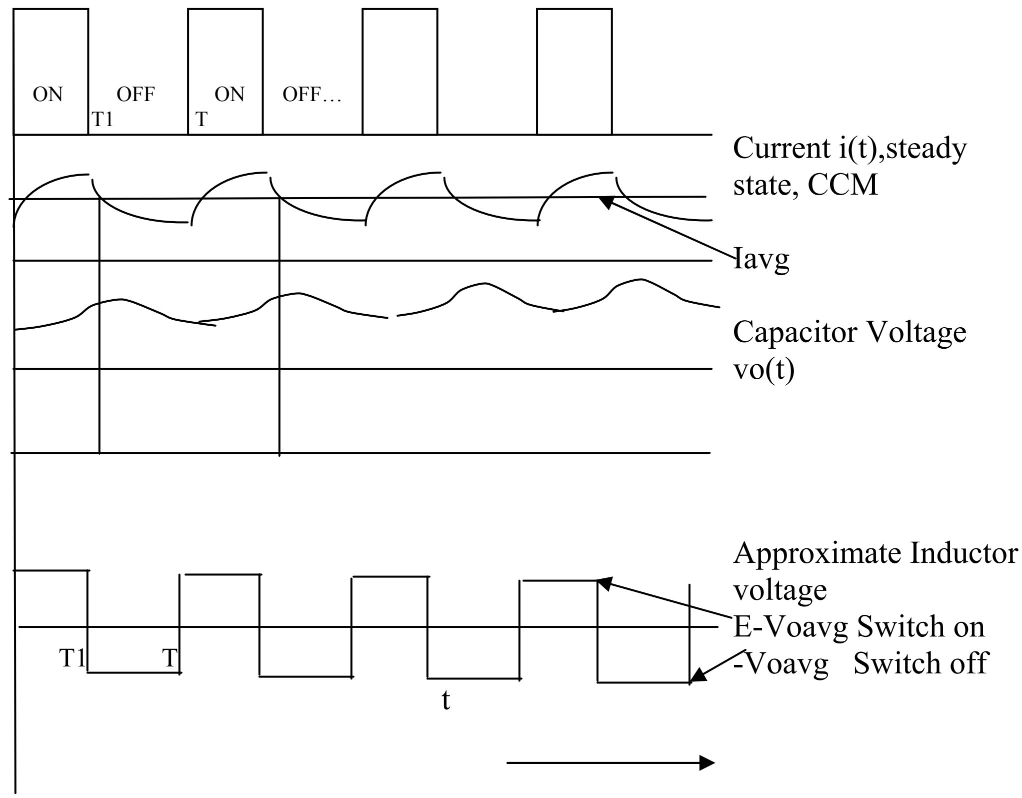
[1] As usual we make reasonable assumptions and charge ahead. If answers turn out inconsistent with the assumption we will need to reconsider.



Circuit Mode 1 –Switch On



Circuit Mode 2 Switch Off



Step2 :

We ignore ripple in vo(t) and approximate it by Voavg

The inductor voltage is

vl(t) = E-Voavg Switch on
vl(t) = -Voavg Switch off

Step 3:

Use the averaging principle

(E-Voavg) T1 + (-Voavg)(T-T1) =0

Thus,

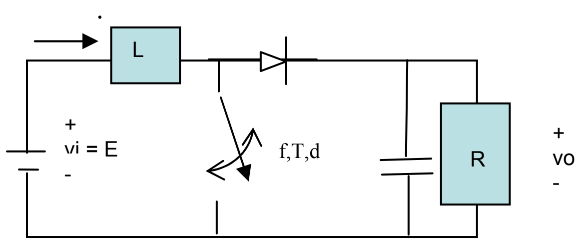
E T1= Voavg T

Voavg = (T1/T) E = D E

Since $0 \leq D \leq 1$, The output voltage is always smaller than source voltage – hence the name step-down or buck (also recall our initial assumption, now found to be consistent)

Example 3 The 'boost' converter

The circuit below is that of a step-down or buck dc-dc converter. The output is a dc voltage. Our goal is to derive a formula for the average output dc voltage

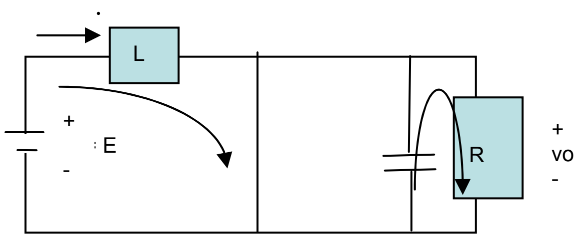


Step 1

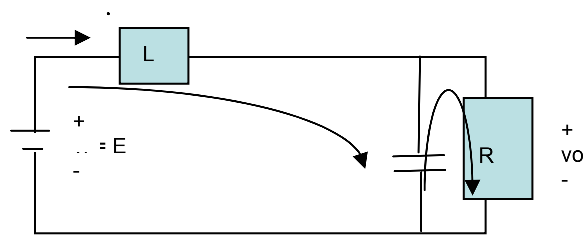
We will first work through circuit operation and sketch waveforms.

When the switch is on (Circuit mode 1) the diode is reverse- biased and off. Current builds up in the inductor. The capacitor supplies load current to the resistor and discharges. Then, when the switch is turned off (circuit mode 2), continuity of inductor current demands that the diode turn on. Inductor current supplies the load and charges the capacitor if this current is greater than load current. Again, both CCM and DCM are possible. We will assume CCM.

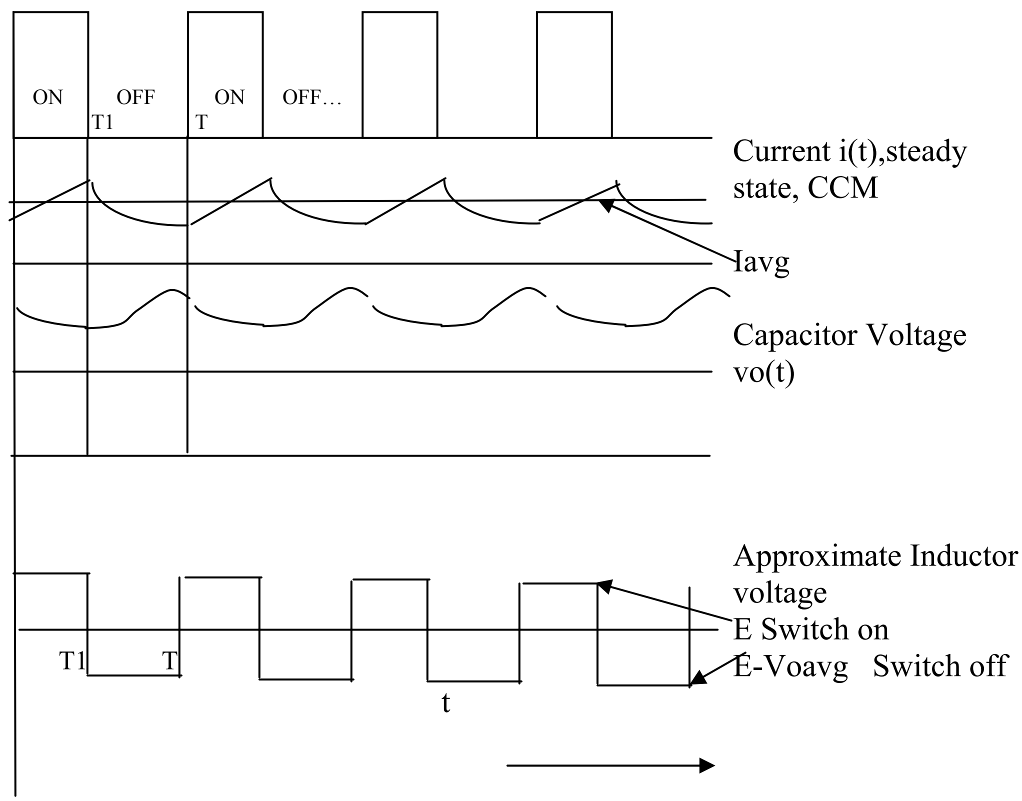
The circuits for each circuit mode and the approximate voltage and current waveforms are shown on the next page.



Circuit Mode 1 –Switch On



Circuit Mode 2 Switch Off



Step2 :

We ignore ripple in vo(t) and approximate it by Voavg

The inductor voltage is

vl(t) = E Switch on

$vl(t) = E - Voavg$ Switch off

Step 3:

Use the averaging principle

$(E)\, T1 + (E - Voavg)(T - T1) = 0$

Thus,

$E\, T = Voavg\, (T - T1) = Voavg\, (1-D)T$

$Voavg = E/(1-D)$

Since $0 \leq D \leq 1$, The output voltage is always larger than source voltage – hence the name step-up or boost.

Example 4 Boost converter

For the boost converter derive the ratio of Input average current to Output average current.

The waveforms of input (inductor) , output and capacitor currents are shown on the next page. The capacitor current can be written as

$ic(t) = io(t)$ Switch on
$ic(t) = i(t) - io(t)$ Switch off

We will ignore ripple and approximate the input current i(t) by its average value Iiavg and the output current io(t) by its average value Ioavg. so

$ic(t) = Ioavg$ Switch on
$ic(t) = Iiavg - Ioavg$ Switch off

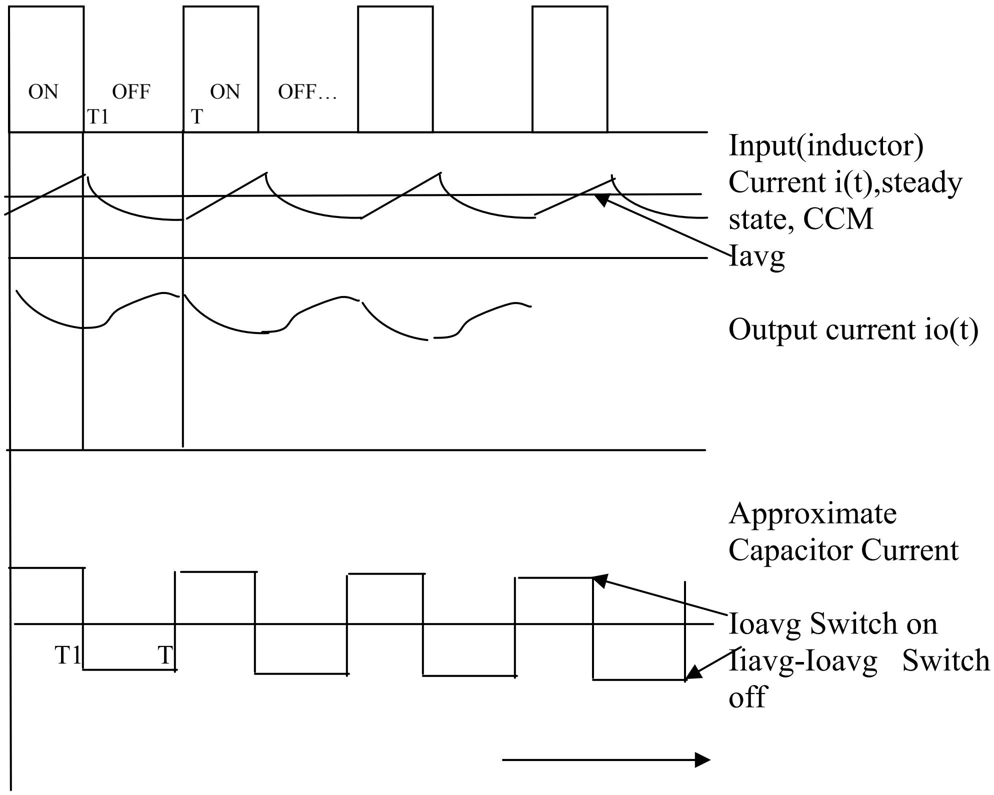
The averaging principle states that the integral of capacitor current over a cycle is zero. Thus

$Ioavg\, T1 + (Iiavg - Ioavg)\, (T - T1) = 0$

$Ioavg\, T + Iiavg\, (T - T1) = 0$

$Ioavg = Iiavg\, (T - T1)/T = Iiavg\, (1-D0)$

The current gain is the reciprocal of voltage gain as it should be!



In contrast to detailed analytic solutions and simulations, the averaging principle allows straightforward computation of the basic properties of periodically switched power electronic circuits. Average values are easily determined as a function of modulation strategy.

The method can be extended to include non-idealities such as resistance by the use of generalized state space averaging as discussed in later sections.