Part 1 A. Analysis and Simulation of Power Electronic Circuits (Continued)

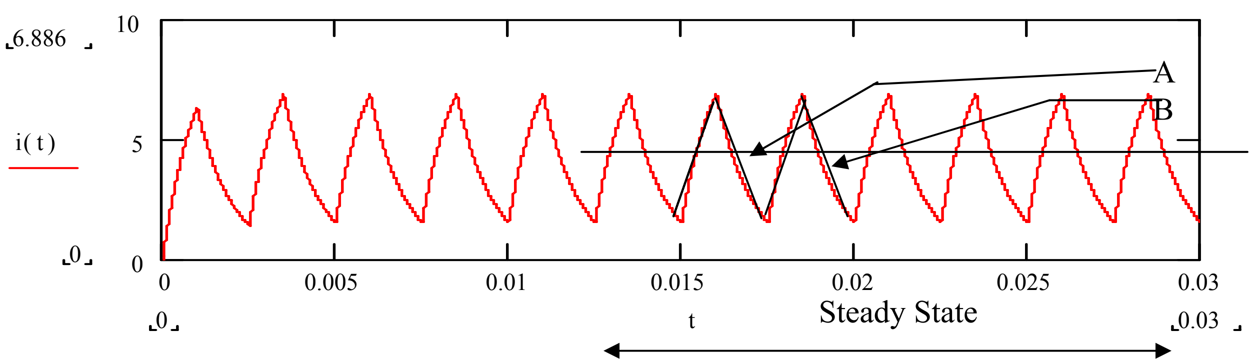
Approximate Analysis of Ripple and CCM/DCM boundary

In the previous lectures we developed a complete analytical solution for periodically switched circuits, specifically, the chopper circuit. Such solutions are at best difficult to obtain.

Next we found that many features of the solution can be very effectively derived using the so-called averaging principle. The most important application of the averaging principle is in deriving formulas for average voltages and currents.

We turn now to examining the ripple around the average in steady-state operation. By making reasonable approximations to circuit waveforms we can readily derive formulas for ripple in terms of component parameters. These formulas can in turn be used for first cut design.

Typical Approximations:
Recall the output current waveform, shown below, for the chopper circuit.



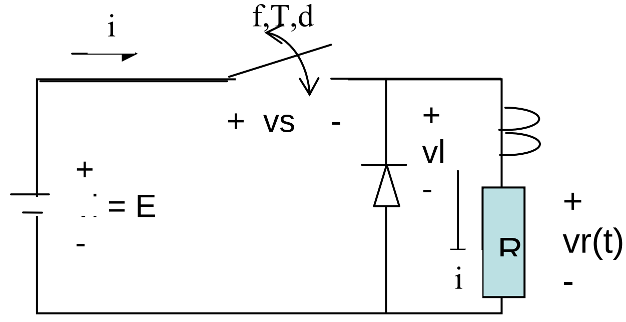
Some common approximations are described below:

A. The zeroth order approximation would replace the current in the steady state region by its average Iavg
B. The first order approximation would model the exponential rise and fall as a triangle wave
C.

Approximation A was used when calculating averages. Approximation B is quite useful when estimating ripple

Example 1: Chopper with inductive filter and free wheeling diode

Recall that the chopper below converts the dc voltage into a dc current whose dc or average value can be controlled by the duty cycle. We wish to determine the average load current

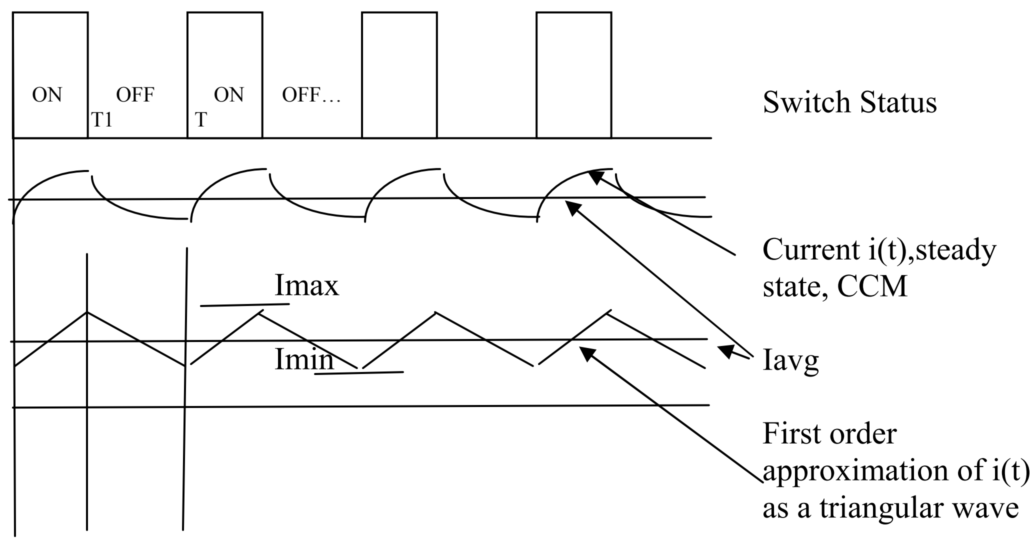


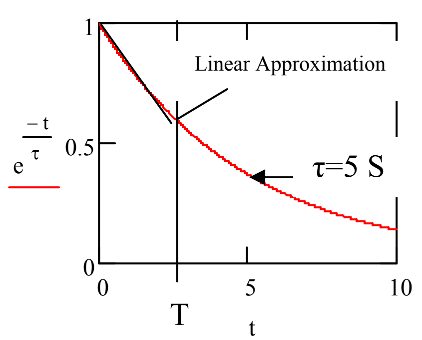
The average current in the load, Iavg, was previously derived to be

Iavg=E D/R

Our goal now is estimate the ripple in the load current.

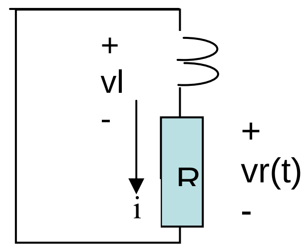
The switch status and waveforms for steady state inductor current and voltage are shown below.





We approximate the exponential rise and decay of output current by a triangle wave. This approximation is reasonable if the time constant $\tau = L/R \gg$ the switching time T.

During the period T1<t<T that the switch is off we are assuming that the current decays linearly.



Since $0 = R\, i(t) + L\, di/dt$

We can write the approximate equation

$$0 \approx R\, I_{avg} + L\, (I_{min} - I_{max})/(T-T1)$$

Where Imin and Imax are the minimum and maximum values of the triangle approximation to i(t)

We see that

$$I_{max} - I_{min} = (T-T1)\, R\, I_{avg}/L$$

Also note that

$$I_{avg} \approx (I_{max} + I_{min})/2$$

Solving for Imax and Imin

$I_{max} = [2+((T-T1)\, R/L)](I_{avg}/2)$
$I_{min} = [2-((T-T1)\, R/L)](I_{avg}/2)$

Thus the ripple in Amperes is

$\Delta I = I_{max} - I_{min} = ((T-T1)\, R/L)\, I_{avg} = [(T-T1)/T]\, (T\, R/L) I_{avg} = (1-D)\, (T/\tau)\, I_{avg}$

Where $\tau = L/R$

Referring to Example 1 with R= 10 ohm, L= 0.03 H and E=100 V, T=1mS and T1=0.4 mS, we get

D= 0.4 $\quad$ $\tau = 0.003$ $\quad$ Iavg=4 A

Imax = 3.6 A $\quad$ Imax=4.4 A $\quad$ $\Delta I = 0.8$ A

$\Delta I/I_{avg} = 20\ \%$
These numbers match the detailed solution very well.

Continuous and Discontinuous Modes

The chopper remains in a continuous conduction mode if Imin > o or, equivalently, $\Delta I < 2$ Iavg.

Thus CCM requires

$(1\text{-}D)\ (T/\tau\ ) < 2$

In our case this translates to

$(1\text{-}D)/3 < 2$

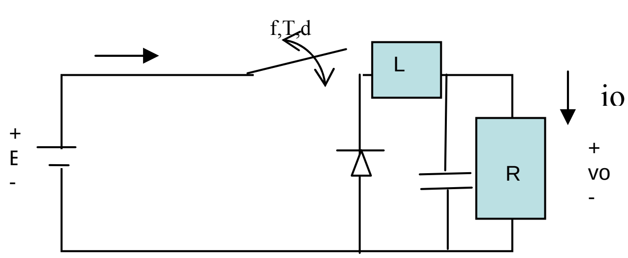
For our parameters CCM is always guaranteed.

However if we reduce inductance to 0.0001H so $T/\tau = 10$, then CCM requires that the duty ratio D>0.8.

Example 2 The buck converter

The buck converter below converts a dc voltage E to a controlled dc voltage Voavg with ripple. Its basic properties are

Voavg = D E
Ioavg = (1/D) Iinavg



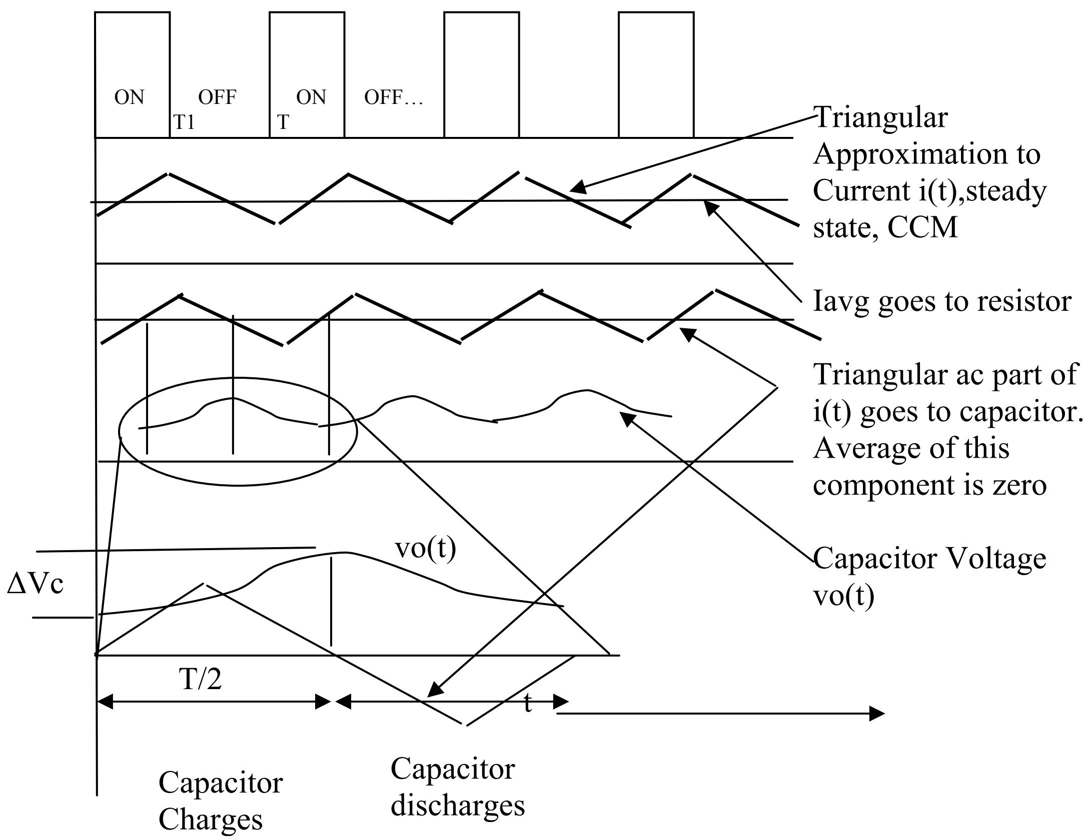
We wish to derive approximate formulas for the ripple in vo, assuming CCM.

We will make the following approximations:

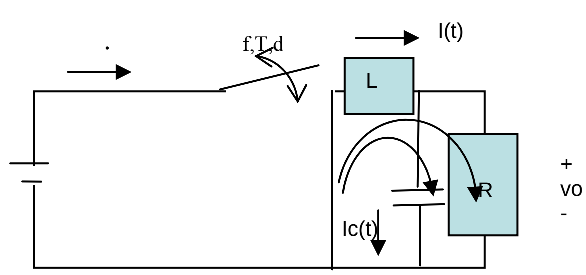
A. The inductor current ripple will be assumed to be a triangle wave.
B. All of the ripple current in the inductor will flow into the capacitor, since the capacitor provides a low impedance path at high frequency

We will first derive a formula for the current ripple, and then one for the voltage ripple.

The basic waveforms for voltage and current in the buck converter are shown below



The derivation of the current ripple formula follows that for the chopper. When the switch is off the circuit is as shown below.



Since $0 = v_o(t) + L\, di/dt$

We can write the approximate equation

$$0 \approx Voavg + L\,(Imin-Imax)/(T-T1) = R\,Iavg + L\,(Imin-Imax)/(T-T1)$$

Where Imin and Imax are the minimum and maximum values of the triangle approximation to i(t)

We see that

$$Imax - Imin = (T-T1)\,Voavg/L = (T-T1)R\,Iavg/L$$

Also note that

$$Iavg \approx (Imax+Imin)/2$$

Solving for Imax and Imin

$Imax = [2+((T-T1)\,R/L)](Iavg/2)$
$Imin = [2-((T-T1)\,R/L)](Iavg/2)$

Thus the ripple in Amperes is

$\Delta I = Imax - Imin = ((T-T1)\,R/L)\,Iavg = = ((1-D)T/L)\,Voavg$

Next consider capacitor voltage. The capacitor charges when the ripple current through it is positive. As seen from the waveforms, the capacitor voltage increases by ΔVc over a period of time T/2. This change can be calculate as follows

$\Delta Vc = \Delta Q/C$

Where ΔQ is the charge deposited in time T/2. This charge in turn is the integral of the positive ripple current.

$\Delta Q = (1/2)\,[(Imax-Imin)/2]\,(T/2)$

Thus $\Delta Vc = (1/8LC)\,(1-D)\,T^2\,Voavg$

Suppose we have a 12 V dc battery and need a 10 V 10 A dc regulated output. The ripple of Vo needs to be $<5\%$. A 1 mH 10 A inductor is available

Let's use a buck switching at 5 kHz and duty D= 10/12 = 0.833

Note R= 10 V/10 A = 1 ohm, T = 0.2 mS and T1= DT = 0.1666 mS

Since $\Delta Vc/Voavg = 0.05$

$C= (1/ 8 * 1 mH)(1-0.833)* (0.0002)^2 = 0.8 uF$

Now let's calculate Imin to make sure the CCM assumption is valid

$Imin = [2-((T-T1) R/L)](Iavg/2) = 9.8 A$