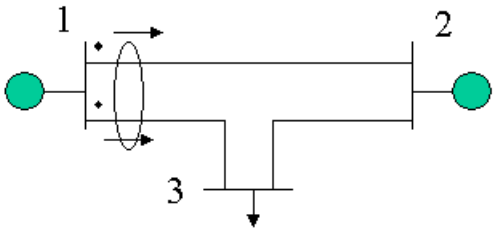
Consider the 3-bus System from Text page 54-55. We are assuming a balanced steady state and working with a per phase model Line impedances given are

For the mutually coupled lines from bus 1-2 and 1-3 let us assu that the primitive impedance matrix denoted by z1213 is



$$z1213 := \begin{pmatrix} .02 + .3i & .05i \\ .05i & .01 + .1i \end{pmatrix}$$

$$z23 := .01 + .1i$$

We will again ignore capacitance for simplicity and assume the following generator impedances

$$zg1 := .1i$$

$$zg2 := .1i$$

z1213 was written in class as z12 M
M z13

The prmitive admittance matrix for just the coupled lines

y1213 was written as y12 m
m y12

$$y1213 := z1213^{-1}$$

$$y1213 = \begin{pmatrix} 0.295 - 3.609i & -0.325 + 1.772i \\ -0.325 + 1.772i & 1.238 - 10.762i \end{pmatrix}$$

$$yprim := \begin{pmatrix} y1213_{1,1} & 0 & y1213_{1,2} & 0 & 0 \\ 0 & \frac{1}{z23} & 0 & 0 & 0 \\ y1213_{2,1} & 0 & y1213_{2,2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{zg1} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{zg2} \end{pmatrix}$$

Since line currents are directed from bus 1 to 2 and bus 1 to 3 for this example

$$C := \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$Y := C^{T} \cdot yprim \cdot C$$

$$Y = \begin{pmatrix} 0.884 - 20.827i & 0.03 + 1.837i & -0.914 + 8.99i \\ 0.03 + 1.837i & 1.285 - 23.51i & -1.315 + 11.673i \\ -0.914 + 8.99i & -1.315 + 11.673i & 2.229 - 20.663i \end{pmatrix}$$

The modified rules for Y are

If line from bus i to bus j is coupled to the line from bus k to bus l and the primitive admittance matrix is

$$Y := \begin{pmatrix} y1 & m \\ m & y2 \end{pmatrix}$$

Add y1 to Yii term
Add y1 to Yjj term
Add y2 to Ykk term
Add y2 to Yll term

Add -y1 to Yij term
Add -y1 to Yji term
Add -y2 to Ykl term
Add -y2 to Ylk term

The above are just the usual rules. In addition

Add m to Yik term
Add m to Yki term
Add m to Yjl term
Add m to Ylj term

Add -m to Yil term
Add -m to Yli term
Add -m to Yjk term
Add -m to Ykj term

So

$$Y := \begin{bmatrix} \left(\frac{1}{zg1}\right) + y1213_{1,1} + y1213_{2,2} + y1213_{1,2} + y1213_{1,2} & -y1213_{1,1} - y1213_{1,2} & -y1213_{2,2} - y1213_{1,2} \\ -y1213_{1,1} - y1213_{1,2} & \left(\frac{1}{zg2}\right) + y1213_{1,1} + \left(\frac{1}{z23}\right) & \left(\frac{-1}{z23}\right) + y1213_{1,2} \\ -y1213_{2,2} - y1213_{1,2} & \left(\frac{-1}{z23}\right) + y1213_{1,2} & \left(\frac{1}{z23}\right) + y1213_{2,2} \end{bmatrix}$$

$$Y = \begin{pmatrix} 0.884 - 20.827i & 0.03 + 1.837i & -0.914 + 8.99i \\ 0.03 + 1.837i & 1.285 - 23.51i & -1.315 + 11.673i \\ -0.914 + 8.99i & -1.315 + 11.673i & 2.229 - 20.663i \end{pmatrix}$$

Fault calculation example

$$Z := Y^{-1} \qquad Z = \begin{pmatrix} 8.975\times10^{-4}+0.071i & -8.975\times10^{-4}+0.029i & -5.492\times10^{-6}+0.047i \\ -8.975\times10^{-4}+0.029i & 8.975\times10^{-4}+0.071i & 5.492\times10^{-6}+0.053i \\ -5.492\times10^{-6}+0.047i & 5.492\times10^{-6}+0.053i & 5.146\times10^{-3}+0.098i \end{pmatrix}$$

Three phase fault at bus 3; prefault voltages = 1 pu

$$If := \frac{1}{Z_{3,3}} \qquad If = 0.532 - 10.156i$$

$$V := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + Z \begin{pmatrix} 0 \\ 0 \\ -If \end{pmatrix} \qquad V = \begin{pmatrix} 0.52 - 0.025i \\ 0.464 - 0.028i \\ 0 \end{pmatrix} \text{ pu}$$

Branch voltage drop $\qquad Vb := C \cdot V \qquad Vb = \begin{pmatrix} 0.056 + 2.801i\times10^{-3} \\ 0.464 - 0.028i \\ 0.52 - 0.025i \\ 0.52 - 0.025i \\ 0.464 - 0.028i \end{pmatrix}$ pu

Branch Current $\qquad Ib := yprim \cdot Vb$

$$Ib = \begin{pmatrix} -0.098 + 0.73i \\ 0.182 - 4.626i \\ 0.35 - 5.53i \\ -0.252 - 5.2i \\ -0.28 - 4.644i \end{pmatrix} \text{ pu}$$

Note :Sum branch 2 and branch 3 currents equals fault current