Lecture 2

This begins the review of sinusoidal, steady state analysis of ac circuit. The basic idea from EE211 is that we can make the analysis straight forward by using phasor methods. Using phasor analysis we will learn to solve some very standard analysis problems in power systems.

Key Ideas

1. Sinusoidal, single-frequency, ac steady analysis of linear, bilateral circuits
    a. Use phasor analysis
    b. Represent sources by phasor, appropriate magnitude and impedance
    c. Represent passive circuit elements by complex impedances at the appropriate frequency
    d. Use KVL, KCL, series/parallel impedance, dividers, Thevenin/Norton…

Text:Section 2.1

Things to remember: In Power systems work Sources are specified in RMS
Don't forget the 'j'! Phasors and impedances are COMPLEX numbers

2. Broader questions

Why are power systems ac? – Transformers, ac Motors,
Why do we/ When can we use phasor analysis?
What is a phasor?

A. Phasor representation defined

$v(t) = V_{max} \cos(\omega t+\delta)=\sqrt{2}\,|V| \cos(\omega t+\delta) = Re(\sqrt{2}\,|V|e^{j\omega t}e^{j\delta})$ ⟵ Time domain form

$|V|\underline{/\delta} = |V|\cos\delta + j\,|V|\sin\delta$ ⟵ Phasor form

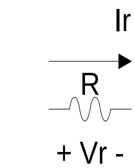
Vmax peak amplitude

|V| rms magnitude ⟵ Standard to give voltage current values in rms

$V_{max}=\sqrt{2}\,|V|$

ω radian frequency $\omega=2\pi f$ f=frequency in hertz ; 60 Hz<> 377 rad/s
δ phase angle with respect to some reference, in radians

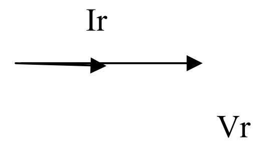
B. Impedance and "Ohms Law" Phasor Diagram

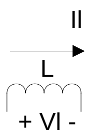


Impedance Zr = R

Vr= Zr Ir = R Ir

Voltage v(t) and current i(t) are in phase

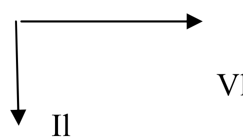


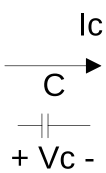


Impedance Zl = jωL

Vl= Zl Il = jωL Il

Voltage v(t) leads current i(t) by 90 deg

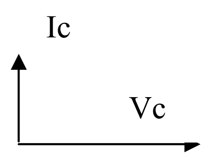




Impedance Zc =1/ jωC = -j /ωC

Vc= Zc Ic =(1/ jωC) Ic

Voltage v(t) lags current i(t) by 90 deg



C. If sinusoidal source with identical frequencies are connected to a circuit with linear bilateral components then, in the steady state, all currents and voltages will have that same frequency. Only the magnitudes and phase angles will be different.

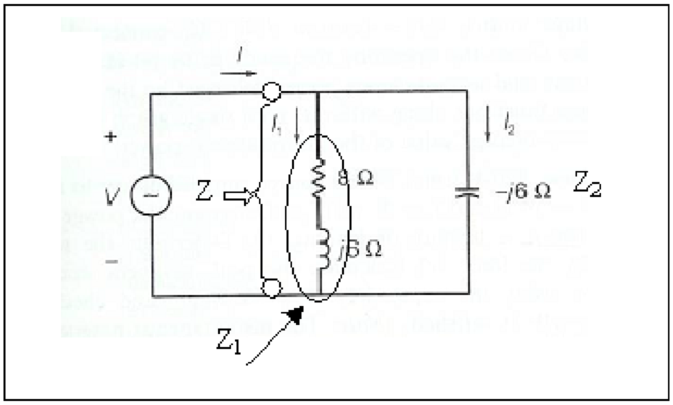
Then instead of differential equation we can use phasor analysis to compute voltages and currents:

- Represent sources by their phasor value(magnitude in rms)
- Represent circuit elements by impedance at the given frequency(Ohm's Law)
- Use appropriate techniques –KVL,KCL, Thevenin, Norton, Series/Parallel..
- Don't forget this is complex arithmetic; don't forget the 'j'

D. Circuit calculations: Example 1

This example was developed in Mathcad. Recall := assigns a value while = displays a result. A complex number has the default form a +b i where i=√-1

2.4 Given I = 10 A find V, I1 and I2. Draw a phasor Diagram. Assume 60 Hz, Steady state



$I := 10 + 0i$ A

$Z_1 := 8 + 6i$ $Z_2 := -6i$ ohms

Approach

Find the equivalent impedance Z seen by thesource
Find V from the given I and calculated Z
Find I1 and I2

The equivalent (parallel) impedance as viewed from the source is

$$Z := Z_1 \cdot \frac{Z_2}{Z_1 + Z_2} \qquad Z = 4.5 - 6i \quad \text{ohms}$$

$$|Z| = 7.5 \qquad \arg(Z) = -53.13\text{deg}$$

Thus

$$V := Z \cdot I \qquad V = 45 - 60i \qquad |V| = 75 \qquad \arg(V) = -53.13\text{deg}$$

$$I_1 := \frac{V}{Z_1} \qquad I_1 = -7.5i \qquad |I_1| = 7.5 \qquad \arg(I_1) = -90\text{deg}$$

$$I_2 := \frac{V}{Z_2} \qquad I_2 = 10 + 7.5i \qquad |I_2| = 12.5 \qquad \arg(I_2) = 36.87\text{deg}$$

Phasor Diagram

The phasor diagram shows the relative orientation of ohasors, in the complex plane. Note I = 10/0deg is along the real (horizontal) axis whilw I_1=7.5/-90deg points vertically downwards

The phasor diagram also allows us to display KCL, KVL and results.

Thus, I is the phasor sum of I_1 and I_2 and lies along the real axis.