Lecture 3

Text: Section 2.2-2.3

Key Ideas-- Power in single phase ac circuits

Instantaneous power $p(t)=v(t)\, i(t)$ always oscillates

The average power corresponds to useful work and is called real power P

Reactive power Q is a measure of the alternating part needed to charge and discharge inductors and capacitors in the ac cycle

Complex power is defined as $S=VI^*$ and equals $P+jQ$

Apparent power is $|S|=|V||I|$ volt-amperes and determines the 'size' of power equipment

Instantaneous power (See p.46-48)

If $v(t) = V_{max} \cos(\omega t+\beta)=\sqrt{2}\,|V| \cos(\omega t+\beta)$ and $i(t) = I_{max} \cos(\omega t+\delta)=\sqrt{2}\,|I| \cos(\omega t+\delta)$

$p(t)=(1/2)V_{max} I_{max}[\cos(\delta-\beta) +\cos\{2(\omega t+\delta)-(\delta-\beta)\}]$

$p(t)= |V||I|\cos\varphi\, [1+\cos\{2(\omega t+\delta)\}] \quad + \quad |V||I|\sin \varphi \sin\{2(\omega t+\delta)\}$

Term 1 Term 2

Where $\phi=\delta-\beta$, the angle by which voltage leads the current, is called the "power factor angle"

Term 1 is oscillating power/energy with average value $P=VI\cos\varphi$.

This term corresponds to net work done or energy provided. P is called real power measured in Watts

The second term is a sinusoidal or alternating component with amplitude $Q=VI \sin \varphi$

but no AVERAGE value, work done, or net energy supply Q is called reactive power measured in VARS. Reactive power is the oscillating energy flow needed to charge/discharge inductors and capacitors in the ac cycle (See Exmple 2.1).

Complex power is defined as $S= VI^* = P+j Q$

Apparent power is $|S|= |V||I|= \sqrt{(P^2+Q^2)}$ VA

Note $|I| = |S|/|V| = \sqrt{(P^2+Q^2)}\ /|V|$

Although Q does not provide any net energy the corresponding oscillating flow has to be maintained if we have L's and C's. The current required depends on P and Q. If you must support lots of Q the current will be high and you need bigger equipment.

Power factor PF= $\cos\varphi = \cos(\delta-\beta)$ = cos(voltage phase-current phase)

Need to tell if Lagging or leading

Resistor> PF = 1 Inductor PF = 0 lag Capacitor PF= 0 lead

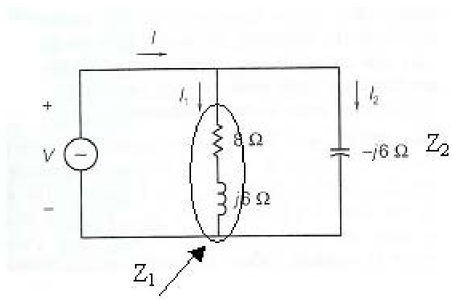
Power calculations $S=VI^*$ is the most convenient

Can use $P= |V||I|\cos\varphi$. $Q= |V||I|\sin\varphi$

H. For impedance Z with Voltage V across, or current I through can use

$S = |I|^2Z = |V|^2/ Z^*$ watch the conjugate

Example Continued from Lecture 2-Power Calculations



$I := 10 + 0i$ A

$Z_1 := 8 + 6i$ $Z_2 := -6i$ ohms

The equivalent (parallel) impedance as viewed from the source is

$$Z := Z_1 \cdot \frac{Z_2}{Z_1 + Z_2} \qquad Z = 4.5 - 6i \quad \text{ohms}$$

$|Z| = 7.5$ $\arg(Z) = -53.13\,\text{deg}$

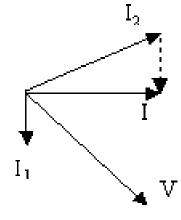
Thus

$V := Z \cdot I$ $V = 45 - 60i$ $|V| = 75$ $\arg(V) = -53.13\,\text{de}$

$I_1 := \frac{V}{Z_1}$ $I_1 = -7.5i$ $|I_1| = 7.5$ $\arg(I_1) = -90\,\text{deg}$

$I_2 := \frac{V}{Z_2}$ $I_2 = 10 + 7.5i$ $|I_2| = 12.5$ $\arg(I_2) = 36.87\,\text{deg}$

Phasor Diagram



Complex Power

Delivered by source

$S_s := V \cdot \overline{I}$ $S_s = 450 - 600i$ VA   450W and – 600Var

Delivered to Z1

$S_1 := V \cdot \overline{I_1}$ $S_1 = 450 + 337.5i$ VA

$S_2 := V \cdot \overline{I_2}$ $S_2 = -937.5i$ VA

Complex Powers and power factor

At source

$\left|S_s\right| = 750$ VA

$pfs := \cos(\arg(V) - \arg(I))$ $pfs = 0.6$ Leading

Load 1

$\left|S_1\right| = 562.5$ VA

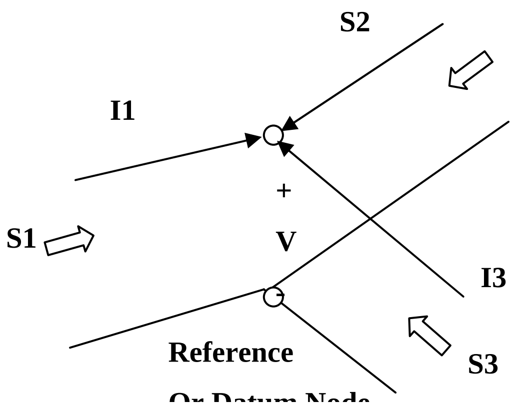
$pf1 := \cos\left(\arg(V) - \arg\left(I_1\right)\right)$ $pf1 = 0.8$ Lag

Load 2

$\left|S_2\right| = 937.5$ VA

$pf2 := \cos\left(\arg(V) - \arg\left(I_2\right)\right)$ $pf2 = 0$ Lead

Complex Power Conservation



KCL I1+I2+I3=0 => V I1* + V I2* + V I3* =0=> S1+S2+S3=0 Complex power conservation

Thus P1+P2+P3 = 0 Q1+Q2+Q3=0

This Mathcad example explores the time variation of instantaneous power. We will look at a poair of voltage and current with different phase relationships and show plots of instantaneous power as wel as the values of corresponding real power reactive power and power factor.

Frequency $\omega := 377$

When plotting in Mathcad we must specify the range of the independent variable and the sample instants for plotting. We will plot zero to 0.05 second, 3 cycles of 60 Hz, at 0.1 ms intervals.

Time range for plotting points every .1 mS $t := 0, .0001 .. .05$

Resistive Load

Let $vl(t) := \sqrt{2} \cdot 1 \cdot \cos(\omega \cdot t)$ $VL := 1 \cdot e^{j \cdot 0deg}$ $il(t) := \sqrt{2} \cdot 1 \cdot \cos(\omega \cdot t)$ $IL := 1 \cdot e^{j \cdot 0deg}$

$VL := 1 \cdot e^{j \cdot 0deg}$ $|VL| = 1$ V $\delta := \arg(VL)$ $\delta = 0deg$

$IL := 1 \cdot e^{j \cdot 0deg}$ $|IL| = 1$ V $\beta := \arg(IL)$ $\beta = 0deg$

$\phi := \delta - \beta$ $\phi = 0$

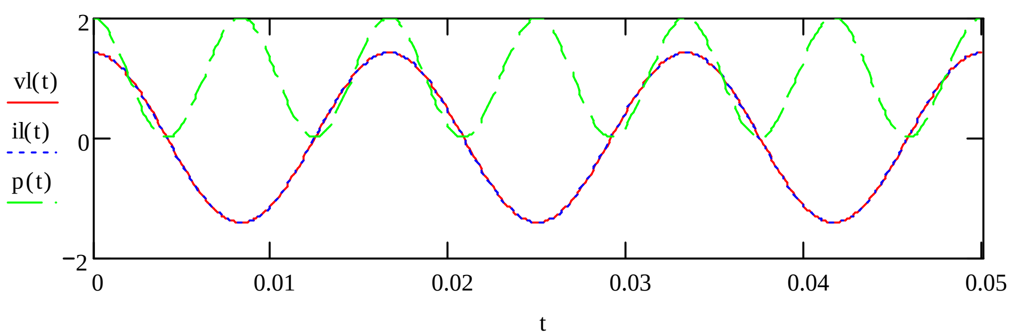
Instantaneous power

Directly $p(t) := 2 \cdot \cos(\omega \cdot t)^2$

Book formula

$$p(t) := |VL| \cdot |IL| \cdot \cos(\phi) \cdot [1 + \cos[2 \cdot (\omega \cdot t + \delta)]] + |VL| \cdot |IL| \cdot \cos(\phi) \cdot \sin[2 \cdot (\omega \cdot t + \delta)]$$

Mathcad $p(t) := vl(t) \cdot il(t)$



Note p(t) oscillates with double frequency and average value of 1; The flow of energy is unidirectional. So we have an oscillation but only real power transfer

Complex power $S := VL \cdot \overline{IL}$ $S = 1$ VA $P := 1$ W $Q := 0$ VAR

Power Factor $PF := \cos(\phi)$ $PF = 1$

Capacitor Load

$vl(t) := \sqrt{2} \cdot 1 \cdot \cos(\omega \cdot t)$ $\qquad$ $il(t) := \sqrt{2} \cdot 1 \cdot \cos(\omega \cdot t + 90deg)$

$VL := 1 \cdot e^{j \cdot 0deg}$ $\quad |VL| = 1$ V $\quad \delta := arg(VL)$ $\quad \delta = 0\,deg$

$IL := 1 \cdot e^{j \cdot 90deg}$ $\quad |IL| = 1$ V $\quad \beta := arg(IL)$ $\quad \beta = 90\,deg$

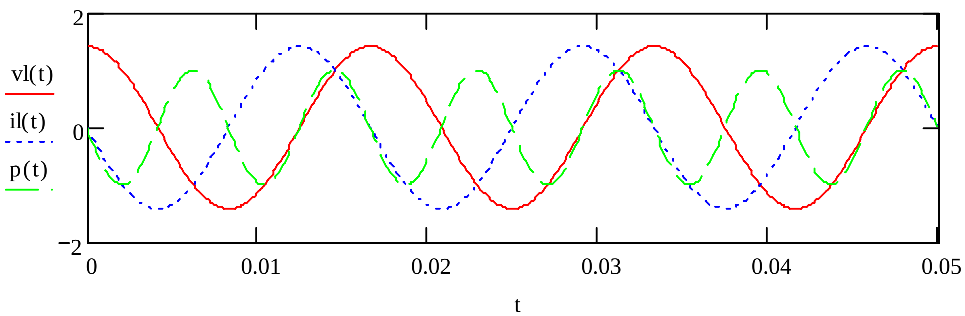
$\phi := \delta - \beta$ $\qquad \phi = -90\,deg$

Instantaneous power

Book formula

$p(t) := |VL| \cdot |IL| \cdot \cos(\phi) \cdot [1 + \cos[2 \cdot (\omega \cdot t + \delta)]] + |VL| \cdot |IL| \cdot \cos(\phi) \cdot \sin[2 \cdot (\omega \cdot t + \delta)]]$

Mathcad $\quad p(t) := vl(t) \cdot il(t)$



Note p(t) oscillates with double frequency and average value of 0; The flow of energy is alternating with zero net energy. As Voltage goes down, capacitor discharges and release energy back into the source. Power flow is negative. Then as voltage decrease and goes more negative, capacitor charges in the negative direction and takes and stores energy from the source. Power and energy flow into the capacitor...

Complex power $\qquad S := VL \cdot \overline{IL}$ $\qquad S = -i$ VA $\quad P := 0$ W $\quad Q := -1$ VAR

Power Factor $\qquad PF := \cos(\phi)$ $\qquad PF = 0$ lead

Inductor Load

$vl(t) := \sqrt{2} \cdot 1 \cdot \cos(\omega \cdot t)$ $VL := 1 \cdot e^{j \cdot 0deg}$ $il(t) := \sqrt{2} \cdot 1 \cdot \cos((\omega \cdot t - 90deg))$ $IL := 1 \cdot e^{j \cdot -90deg}$

$VL := 1 \cdot e^{j \cdot 0deg}$ $|VL| = 1$ V $\delta := \arg(VL)$ $\delta = 0\,deg$

$IL := 1 \cdot e^{j \cdot -90deg}$ $|IL| = 1$ V $\beta := \arg(IL)$ $\beta = -90\,deg$

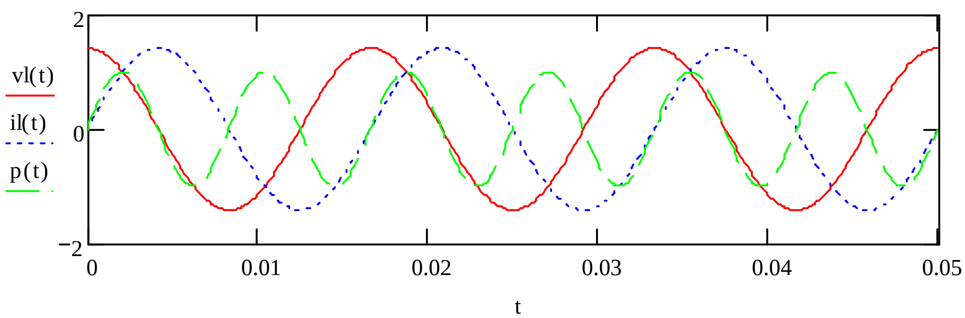
$\phi := \delta - \beta$ $\phi = 1.571$ Voltage leads current

Instantaneous power

Book formula

$p(t) := |VL| \cdot |IL| \cdot \cos(\phi) \cdot [1 + \cos[2 \cdot (\omega \cdot t + \delta)]] + |VL| \cdot |IL| \cdot \cos(\phi) \cdot \sin[2 \cdot (\omega \cdot t + \delta)]]$

Mathcad $p(t) := vl(t) \cdot il(t)$



Note p(t) oscillates with double frequency and average value of 0; The flow of energy is alternating with zero net energy. As current goes up, so does the magnetic field and energy stored in the inductor. Power and Energy flow is positive. Then ascurrent decreases and energy is released and power flows back to the source

Complex power $S := VL \cdot \overline{IL}$ $S = i$ VA $P := 0$ W $Q := 1$ VAR

Power Factor $PF := \cos(\phi)$ $PF = 0$ lag

General case

$vl(t) := \sqrt{2}\cdot 1\cdot \cos(\omega\cdot t)$ $il(t) := \sqrt{2}\cdot 1\cdot \cos(\omega\cdot t - 30deg)$

$VL := 1\cdot e^{j\cdot 0deg}$ $IL := 1\cdot e^{j\cdot -30deg}$

$|VL| = 1$ V $\delta := arg(VL)$ $\delta = 0\,deg$

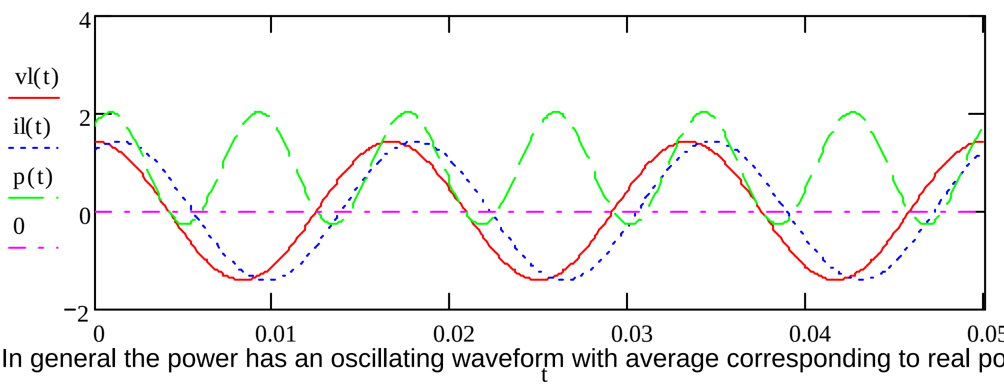
$|IL| = 1$ V $\beta := arg(IL)$ $\beta = -30\,deg$

$\phi := \delta - \beta$ $\phi = 30\,deg$

Instantaneous power

Book formula

$p(t) := |VL|\cdot|IL|\cdot\cos(\phi)\cdot[1+\cos[2\cdot(\omega\cdot t+\delta)] + |VL|\cdot|IL|\cdot\cos(\phi)\cdot\sin[2\cdot(\omega\cdot t+\delta)]]$



In general the power has an oscillating waveform with average corresponding to real power

Complex power $S := VL\cdot\overline{IL}$ $S = 0.866 + 0.5i$ VA $P := 0.866$ W $Q := .5$ VAR

Power Factor $PF := \cos(\phi)$ $PF = 0.866$ lag