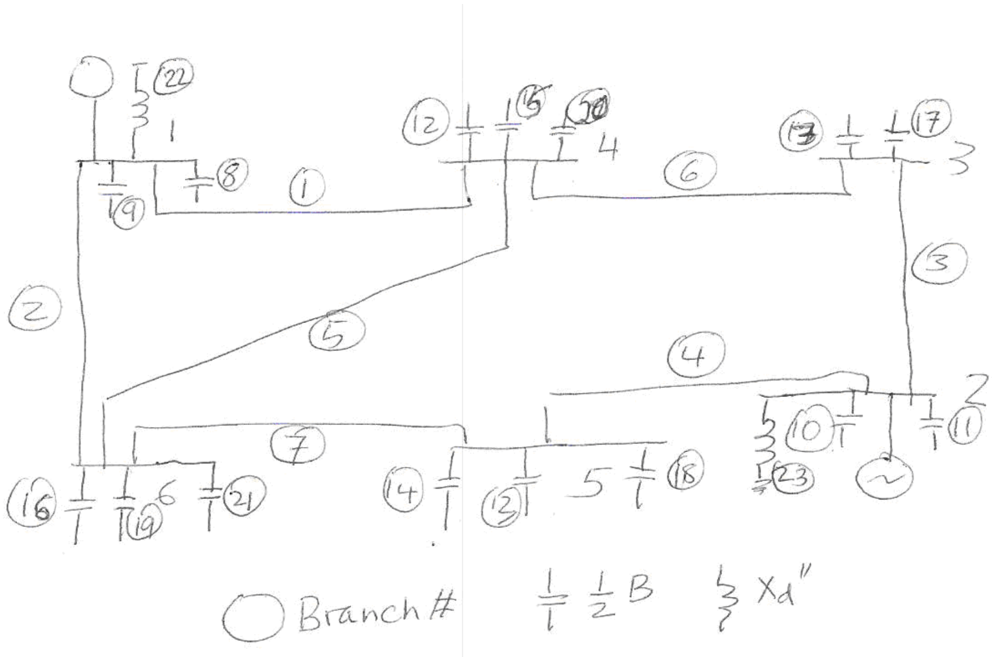
The system can be modeled as 23 branches-- 7 lines, fourteen shunts (Pi model, half shunt admittance at each end) and 2 generator subtransient impedances to ground.



Tooo lazy to type 23x23 matrix.   I will set up vectors with to and from buses for each branch and then use a loop to build connection matrix. Later these will lead to 'linked-lists' for sparse operations

$$From := (1\ \ 1\ \ 2\ \ 2\ \ 4\ \ 3\ \ 5\ \ 1\ \ 1\ \ 2\ \ 2\ \ 4\ \ 3\ \ 5\ \ 4\ \ 6\ \ 3\ \ 5\ \ 6\ \ 4\ \ 6\ \ 1\ \ 2)^T$$

$$To := (4\ \ 6\ \ 3\ \ 5\ \ 6\ \ 4\ \ 6\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)^T$$

zero out C matrix

$i := 1, 2 .. 23$ $j := 1, 2 .. 6$ $C_{i, j} := 0$

Build C using From and To lists

$j := 1, 2 .. 23$ $C_{j, From_j} := 1$

$j := 1, 2 .. 7$ $C_{j, To_j} := \begin{cases} 0 & \text{if } To_j = 0 \\ -1 & \text{otherwise} \end{cases}$

C =

	1	2	3	4	5	6
1	1	0	0	-1	0	0
2	1	0	0	0	0	-1
3	0	1	-1	0	0	0
4	0	1	0	0	-1	0
5	0	0	0	1	0	-1
6	0	0	1	-1	0	0
7	0	0	0	0	1	-1
8	1	0	0	0	0	0
9	1	0	0	0	0	0
10	0	1	0	0	0	0
11	0	1	0	0	0	0
12	0	0	0	1	0	0
13	0	0	1	0	0	0
14	0	0	0	0	1	0
15	0	0	0	1	0	0
16	0	0	0	0	0	1
17	0	0	1	0	0	0
18	0	0	0	0	1	0
19	0	0	0	0	0	1
20	0	0	0	1	0	0
21	0	0	0	0	0	1
22	1	0	0	0	0	0
23	0	1	0	0	0	0

Primitive admittance matrix for 23 branches

$m := 1, 2 .. 23 \quad n := 1, 2 .. 23 \qquad y_{m,n} := 0$

Line series impedance

$y_{1,1} := \frac{1}{.02 + .185i} \qquad y_{2,2} := \frac{1}{.031 + .259i} \qquad y_{3,3} := \frac{1}{.006 + .025i}$

$y_{4,4} := \frac{1}{.071 + .32i} \qquad y_{5,5} := \frac{1}{.024 + .204i} \qquad y_{6,6} := \frac{1}{.075 + .067i}$

$y_{7,7} := \frac{1}{.025 + .15i}$

Half of line shunt admittance, one at each terminal bus for the line

$y_{8,8} := \frac{.009i}{2} \qquad y_{9,9} := \frac{.01i}{2} \qquad y_{10,10} := 0$

$y_{11,11} := \frac{.015i}{2} \qquad y_{12,12} := \frac{.01i}{2} \qquad y_{13,13} := 0 \qquad y_{14,14} := \frac{.017i}{2}$

Generator subtransient

$y_{22,22} := \frac{1}{.1i} \qquad y_{23,23} := \frac{1}{.1i}$

$m := 15, 16 .. 21 \; y_{m,m} := y_{m-7,m-7}$ Repeat other half of susceptance

Compute admittance Matrix

$Y := C^T \cdot y \cdot C$

$$\mathrm{Re}(Y) = \begin{pmatrix} 1.033 & 0 & 0 & -0.578 & 0 & -0.456 \\ 0 & 9.738 & -9.077 & 0 & -0.661 & 0 \\ 0 & -9.077 & 16.493 & -7.415 & 0 & 0 \\ -0.578 & 0 & -7.415 & 8.562 & 0 & -0.569 \\ 0 & -0.661 & 0 & 0 & 1.742 & -1.081 \\ -0.456 & 0 & 0 & -0.569 & -1.081 & 2.106 \end{pmatrix}$$

$$\mathrm{Im}(Y) = \begin{pmatrix} -19.14 & 0 & 0 & 5.343 & 0 & 3.806 \\ 0 & -50.792 & 37.821 & 0 & 2.978 & 0 \\ 0 & 37.821 & -44.446 & 6.624 & 0 & 0 \\ 5.343 & 0 & 6.624 & -16.793 & 0 & 4.835 \\ 0 & 2.978 & 0 & 0 & -9.449 & 6.486 \\ 3.806 & 0 & 0 & 4.835 & 6.486 & -15.109 \end{pmatrix}$$

Using Rules

In the PDF the right half of the matrix will appear on the last page

$$YR := \begin{pmatrix} y_{1,1} + y_{2,2} + y_{8,8} + y_{9,9} + y_{22,22} & 0 & 0 \\ 0 & y_{3,3} + y_{4,4} + y_{23,23} + y_{10,10} + y_{11,11} & -y_{3,} \\ 0 & -y_{3,3} & y_{3,3} + y_{6,6} + y_{1} \\ -y_{1,1} & 0 & -y_{6,} \\ 0 & -y_{4,4} & 0 \\ -y_{2,2} & 0 & 0 \end{pmatrix}$$

$$\mathrm{Re}(YR) = \begin{pmatrix} 1.033 & 0 & 0 & -0.578 & 0 & -0.456 \\ 0 & 9.738 & -9.077 & 0 & -0.661 & 0 \\ 0 & -9.077 & 16.493 & -7.415 & 0 & 0 \\ -0.578 & 0 & -7.415 & 8.562 & 0 & -0.569 \\ 0 & -0.661 & 0 & 0 & 1.742 & -1.081 \\ -0.456 & 0 & 0 & -0.569 & -1.081 & 2.106 \end{pmatrix}$$

$$\mathrm{Im}(YR) = \begin{pmatrix} -19.14 & 0 & 0 & 5.343 & 0 & 3.806 \\ 0 & -50.792 & 37.821 & 0 & 2.978 & 0 \\ 0 & 37.821 & -44.446 & 6.624 & 0 & 0 \\ 5.343 & 0 & 6.624 & -16.793 & 0 & 4.835 \\ 0 & 2.978 & 0 & 0 & -9.449 & 6.486 \\ 3.806 & 0 & 0 & 4.835 & 6.486 & -15.109 \end{pmatrix}$$

Comparing the two they are the same, with slight differences in the 3rd decimal place(Why??)

Tryong to automate the construction of Y in Mathcad is a little clumsy, since I took the array origin as 1.

First I will define the refrence (usually node 0) as node 7. Then I will use the rules to build a 7x7 matrix. Finally I will throw away row and column 7

$$From := (1\ \ 1\ \ 2\ \ 2\ \ 4\ \ 3\ \ 5\ \ 1\ \ 1\ \ 2\ \ 2\ \ 4\ \ 5\ \ 4\ \ 6\ \ 3\ \ 5\ \ 4\ \ 6\ \ 4\ \ 6\ \ 1\ \ 2)^T$$

$$To := (4\ \ 6\ \ 3\ \ 5\ \ 6\ \ 4\ \ 6\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7\ \ 7)^T$$

$m := 1, 2 .. 7$ $n := 1, 2 .. 7$ $\quad Y7_{m,n} := 0$

$m := 1, 2 .. 23$

$$Y7_{From_m, From_m} := Y7_{From_m, From_m} + y_{m,m}$$

$$Y7_{To_m, To_m} := Y7_{To_m, To_m} + y_{m,m}$$

$$Y7_{From_m, To_m} := Y7_{From_m, To_m} - y_{m,m}$$

$$Y7_{To_m, From_m} := Y7_{To_m, From_m} - y_{m,m}$$

$$Y := \text{submatrix}(Y7, 1, 6, 1, 6)$$

$$Re(Y) = \begin{pmatrix} 1.033 & 0 & 0 & -0.578 & 0 & -0.456 \\ 0 & 9.738 & -9.077 & 0 & -0.661 & 0 \\ 0 & -9.077 & 16.493 & -7.415 & 0 & 0 \\ -0.578 & 0 & -7.415 & 8.562 & 0 & -0.569 \\ 0 & -0.661 & 0 & 0 & 1.742 & -1.081 \\ -0.456 & 0 & 0 & -0.569 & -1.081 & 2.106 \end{pmatrix}$$

$$Im(Y) = \begin{pmatrix} -19.14 & 0 & 0 & 5.343 & 0 & 3.806 \\ 0 & -50.792 & 37.821 & 0 & 2.978 & 0 \\ 0 & 37.821 & -44.441 & 6.624 & 0 & 0 \\ 5.343 & 0 & 6.624 & -16.781 & 0 & 4.835 \\ 0 & 2.978 & 0 & 0 & -9.465 & 6.486 \\ 3.806 & 0 & 0 & 4.835 & 6.486 & -15.11 \end{pmatrix}$$

Coupled case

with coupling

coupling impedance

admittance

$$zc := \begin{pmatrix} .071 + .32i & .03i \\ .03i & .025 + .15i \end{pmatrix} \qquad yc := zc^{-1} \qquad yc = \begin{pmatrix} 0.693 - 3.024i & -0.233 + 0.566i \\ -0.233 + 0.566i & 1.145 - 6.589i \end{pmatrix}$$

The following terms in the branch primitive admittance will change

Branch 2-5 is #4 and branch 2-6 is #7

$y_{4,4} := yc_{1,1}$ $\qquad y_{4,7} := yc_{1,2}$ $\qquad y_{7,4} := yc_{2,1}$ $\qquad y_{7,7} := yc_{2,2}$ $\qquad y_{4,4} = 0.693 - 3.024i$

$YM := C^T \cdot y \cdot C$

$$Re(YM) = \begin{pmatrix} 1.033 & 0 & 0 & -0.578 & 0 & -0.456 \\ 0 & 9.77 & -9.077 & 0 & -0.926 & 0.233 \\ 0 & -9.077 & 16.493 & -7.415 & 0 & 0 \\ -0.578 & 0 & -7.415 & 8.562 & 0 & -0.569 \\ 0 & -0.926 & 0 & 0 & 2.303 & -1.378 \\ -0.456 & 0.233 & 0 & -0.569 & -1.378 & 2.169 \end{pmatrix}$$

$$\mathrm{Im}(YM) = \begin{pmatrix} -19.14 & 0 & 0 & 5.343 & 0 & 3.806 \\ 0 & -50.838 & 37.821 & 0 & 3.59 & -0.566 \\ 0 & 37.821 & -44.446 & 6.624 & 0 & 0 \\ 5.343 & 0 & 6.624 & -16.793 & 0 & 4.835 \\ 0 & 3.59 & 0 & 0 & -10.73 & 7.155 \\ 3.806 & -0.566 & 0 & 4.835 & 7.155 & -15.212 \end{pmatrix}$$

Using rules

Set admittance parameter for line 2-5 and 5-6 to zero and build Y as usual

$y_{4,7} := 0 \qquad y_{7,4} := 0$

$m := 1, 2..7 \quad n := 1, 2..7 \qquad Y7_{m,n} := 0$

$m := 1, 2..23$

$Y7_{From_m, From_m} := Y7_{From_m, From_m} + y_{m,m}$

$Y7_{To_m, To_m} := Y7_{To_m, To_m} + y_{m,m}$

$Y7_{From_m, To_m} := Y7_{From_m, To_m} - y_{m,m}$

$Y7_{To_m, From_m} := Y7_{To_m, From_m} - y_{m,m}$

$YMR := \mathrm{submatrix}(Y7, 1, 6, 1, 6)$

Now put in coupled branches using modified rules. In terms of class note
i=2 j= 5 k= 5 l=6

$m := yc_{1,2}$

$YMR_{2,5} := YMR_{2,5} + m \qquad YMR_{5,2} := YMR_{5,2} + m$

$YMR_{6,5} := YMR_{6,5} + m \qquad YMR_{5,6} := YMR_{5,6} + m$

$YMR_{6,2} := YMR_{6,2} - m$ $\quad$ $YMR_{2,6} := YMR_{2,6} - m$

$YMR_{5,5} := YMR_{5,5} - m$ $\quad$ $YMR_{5,5} := YMR_{5,5} - m$

$$Re(YMR) = \begin{pmatrix} 1.033 & 0 & 0 & -0.578 & 0 & -0.456 \\ 0 & 9.77 & -9.077 & 0 & -0.926 & 0.233 \\ 0 & -9.077 & 16.493 & -7.415 & 0 & 0 \\ -0.578 & 0 & -7.415 & 8.562 & 0 & -0.569 \\ 0 & -0.926 & 0 & 0 & 2.303 & -1.378 \\ -0.456 & 0.233 & 0 & -0.569 & -1.378 & 2.169 \end{pmatrix}$$

$$Re(YM) = \begin{pmatrix} 1.033 & 0 & 0 & -0.578 & 0 & -0.456 \\ 0 & 9.77 & -9.077 & 0 & -0.926 & 0.233 \\ 0 & -9.077 & 16.493 & -7.415 & 0 & 0 \\ -0.578 & 0 & -7.415 & 8.562 & 0 & -0.569 \\ 0 & -0.926 & 0 & 0 & 2.303 & -1.378 \\ -0.456 & 0.233 & 0 & -0.569 & -1.378 & 2.169 \end{pmatrix}$$

$$Im(YM) = \begin{pmatrix} -19.14 & 0 & 0 & 5.343 & 0 & 3.806 \\ 0 & -50.838 & 37.821 & 0 & 3.59 & -0.566 \\ 0 & 37.821 & -44.446 & 6.624 & 0 & 0 \\ 5.343 & 0 & 6.624 & -16.793 & 0 & 4.835 \\ 0 & 3.59 & 0 & 0 & -10.73 & 7.155 \\ 3.806 & -0.566 & 0 & 4.835 & 7.155 & -15.212 \end{pmatrix}$$

$$Im(YMR) = \begin{pmatrix} -19.14 & 0 & 0 & 5.343 & 0 & 3.806 \\ 0 & -50.838 & 37.821 & 0 & 3.59 & -0.566 \\ 0 & 37.821 & -44.441 & 6.624 & 0 & 0 \\ 5.343 & 0 & 6.624 & -16.781 & 0 & 4.835 \\ 0 & 3.59 & 0 & 0 & -10.746 & 7.155 \\ 3.806 & -0.566 & 0 & 4.835 & 7.155 & -15.213 \end{pmatrix}$$

These match

Fault at bus 5

Uncoupled case $Z := Y^{-1}$ Bus Impedance Matrix

Three phase fault at bus 5 Assume prefault voltages of 1 pu

$$If'' := \frac{1}{Z_{5,5}}$$ $If'' = 0.667 - 4.854i$ Fault current

$i := 1, 2 .. 6$ $$V_i := 1 - \frac{Z_{i,5}}{Z_{5,5}}$$

$$V_i = \begin{pmatrix} 0.788 - 0.02i \\ 0.725 - 0.047i \\ 0.71 - 0.047i \\ 0.662 - 0.013i \\ 0 \\ 0.408 - 0.017i \end{pmatrix}$$ Voltages

$Ib := y \cdot C \cdot V$ $i := 1, 2 .. 7$

$\left\|Ib_i\right\| =$	$From_i =$	$To_i =$
0.675	1	4
1.456	1	6
0.578	2	3
2.253	2	5
1.239	4	6
0.576	3	4
2.73	5	6

$i := 22, 23 .. 23$

$\left\|Ib_i\right\| =$	$From_i =$	$To_i =$
7.88	1	7
7.261	2	7

Coupled case $Z := YM^{-1}$ Bus Impedance Matrix

Three phase fault at bus 5 Assume prefault voltages of 1 pu

$If'' := \frac{1}{Z_{5,5}}$ $If'' = 0.776 - 5.209i$ Fault current

$i := 1, 2 .. 6$ $V_i := 1 - \frac{Z_{i,5}}{Z_{5,5}}$

$$V_i = \begin{pmatrix} 0.772 - 0.024i \\ 0.703 - 0.054i \\ 0.688 - 0.054i \\ 0.637 - 0.019i \\ 0 \\ 0.365 - 0.026i \end{pmatrix}$$

Volatages

$IbM := y \cdot C \cdot V$ $i := 1, 2 .. 7$

$\lvert IbM_i \rvert =$	$From_i =$	$To_i =$
0.726	1	4
1.56	1	6
0.607	2	3
2.188	2	5
1.325	4	6
0.607	3	4
2.45	5	6

$i := 22, 23 .. 23$

$\lvert IbM_i \rvert =$	$From_i =$	$To_i =$
7.729	1	7
7.052	2	7

Compare

$i := 1, 2 .. 7$

$\left\|IbM_i\right\| =$	$\left\|Ib_i\right\| =$	$From_i =$	$To_i =$
0.726	0.675	1	4
1.56	1.456	1	6
0.607	0.578	2	3
2.188	2.253	2	5
1.325	1.239	4	6
0.607	0.576	3	4
2.45	2.73	5	6

Big difference in Line 5-6 which is coupled & close to fault!

$-y_{1,1}$ 0

3 0 $-y_{4,4}$

$0,10 + y_{13,13}$ $-y_{6,6}$ 0

6 $y_{1,1} + y_{6,6} + y_{5,5} + y_{8,8} + y_{12,12} + y_{13,13}$ 0

0 $y_{4,4} + y_{7,7} + y_{11,11} + y_{14,14}$

$-y_{5,5}$ $-y_{7,7}$ $y_{2,2} + y_{5,}$

$$
\begin{matrix}
-y_{2,2} \\
0 \\
0 \\
-y_{5,5} \\
-y_{7,7} \\
{}_{,5} + y_{7,7} + y_{9,9} + y_{12,12} + y_{14,14}
\end{matrix}\right)
$$