

Compensation

Now let's try a 300 mile line. Both voltage drop and rise problems will be exacerbated and we will need to compensate the line to make it work

Long line

$$\begin{aligned}
 \underline{z} &:= 0.017 + 0.483i \quad \frac{\text{ohm}}{\text{mi}} & \underline{y} &:= 8.59i \times 10^{-6} \quad \frac{\text{mho}}{\text{mi}} & l &:= 300 \text{ mi} \\
 \underline{\gamma} &:= \sqrt{\underline{z} \cdot \underline{y}} & \underline{Z_c} &:= \sqrt{\frac{\underline{z}}{\underline{y}}} & \gamma &= 3.584 \times 10^{-5} + 2.037i \times 10^{-3} & Z_c &= 237.118 \\
 \underline{Z} &:= 300 \cdot \underline{z} \quad \text{ohm} & \underline{Y} &:= 300 \cdot \underline{y} & & & & \text{mho} \\
 \underline{A} &:= \cosh(\gamma \cdot l) & \underline{B} &:= Z_c \cdot \sinh(\gamma \cdot l) & \underline{C} &:= \frac{\sinh(\gamma \cdot l)}{Z_c} & \underline{D} &:= A \\
 A &= 0.819 + 6.17i \times 10^{-3} & B &= 4.483 + 136.06i & C &= -5.437 \times 10^{-6} + 2.42i \times 10^{-3} \\
 B &= 4.483 + 136.06i \\
 \underline{Y}' &:= \frac{2 \cdot \tanh\left(\gamma \cdot \frac{l}{2}\right)}{Z_c} & \underline{Z}' &:= Z_c \cdot \sinh(\gamma \cdot l) \\
 Y' &= 3.046 \times 10^{-6} + 2.66i \times 10^{-3} & Z' &= 4.483 + 136.06i
 \end{aligned}$$

Step 3 Performance of uncompensated line

Full and no load ABCD matrix

$$\text{ABCDline} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad \text{ABCD} := \text{ABCDline}$$

Step 3A Full Load Conditions

$$\begin{aligned}
 \underline{V_{Rfl}} &:= \frac{500000}{\sqrt{3}} \quad \text{V} & \underline{I_{Rfl}} &:= \underline{I_R} & \underline{I_R} &= 692.82 - 22.4719i \\
 |\underline{V_{Rfl}}| &= 2.887 \times 10^5 \\
 \underline{V_s} &:= A \cdot \underline{V_{Rfl}} + B \cdot \underline{I_{Rfl}} & |\underline{V_s}| &= 2.867 \times 10^5 \quad 1 - n & \arg(\underline{V_s}) &= 19.355 \text{ deg} \\
 \underline{I_s} &:= C \cdot \underline{V_{Rfl}} + D \cdot \underline{I_{Rfl}} & |\underline{I_s}| &= 767.01 & \arg(\underline{I_s}) &= 42.303 \text{ deg}
 \end{aligned}$$

$$\underline{V_{ml}} := \frac{\underline{V_s}}{A} \quad |\underline{V_{ml}}| = 3.501 \times 10^5 \quad 1 - n \quad \sqrt{3} \cdot |\underline{V_s}| = 4.966 \times 10^5 \quad \text{V L-L}$$

$$SSfl := 3 \cdot V_s \cdot \bar{I}_s \quad SSfl = 6.075 \times 10^8 - 2.572i \times 10^8 \quad \text{Sending end power}$$

$$\text{Losses} \quad S_{\text{loss}} := (V_s \cdot \bar{I}_s - VRfl \cdot \overline{IRfl}) \cdot 3 \quad S_{\text{loss}} = 7.548 \times 10^6 - 4.545i \times 10^8$$

Step 3b No load analysis

$$VRnl := \frac{V_s}{A} \quad |VRnl| = 3.501 \times 10^5 \quad \sqrt{3} \cdot |Vrnl| = 6.063 \times 10^5$$

$$Isnl := C \cdot VRnl \quad |Isnl| = 847.03$$

Step 3c Voltage regulation

$$Reg := \frac{(|VRnl| - |VRfl|) \cdot (100)}{|VRfl|} \quad Reg = 21.268 \quad \text{percent}$$

Step 3d Loadability at $\delta=35$ deg Remember, we use rated values for voltages

$$V_s := \frac{500000}{\sqrt{3}} \quad V_r := \frac{500000}{\sqrt{3}}$$

$$P_{\text{max}} := \left[|V_s| \cdot \frac{|V_r| \cdot \cos\left(\arg(B) - 90 \cdot \frac{3.14}{180}\right)}{|B|} - (|V_r|)^2 \cdot \text{Re}\left(\frac{A}{B}\right) \right] \cdot 3 \quad P_{\text{max}} = 1.775 \times 10^9$$

$$P_{\text{maxpractical}} := \left[|V_s| \cdot \frac{|V_r| \cdot \cos\left(\arg(B) - 35 \cdot \frac{3.14}{180}\right)}{|B|} - (|V_r|)^2 \cdot \text{Re}\left(\frac{A}{B}\right) \right] \cdot 3$$

$$P_{\text{maxpractical}} = 1.041 \times 10^9$$

Regulation is unacceptable

- Full Load VS is OK
- No Load voltage jumps to 600 kV -- this is a problem

Practical Pmax at $\delta=35$ deg is too close to the desired power transfer of 600 MW

Sending end reactive power at full load is 257 MVAR leading -- this is bad

Compensation

Series compensation

Reduce overall series inductive impedance

Reduce full load drop

Increase loadability

Definition

% Compensation = $100 X_c/X_l$

Limits

Voltage across capacitor during short circuit
Subsynchronous resonance

Keep < 70 %

Connection

One end or other

Middle

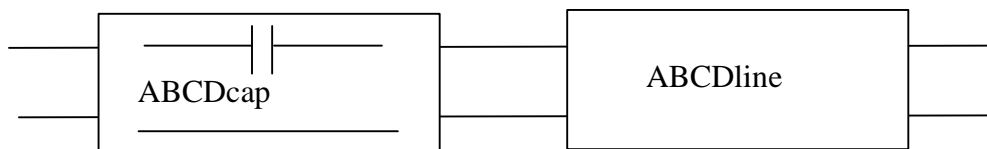
Split each end

Often dictated by substation availability

Example

50 % compensation at sending end

$X_c := 0.5 \cdot \text{Im}(Z)$ $X_c = 72.45 \text{ ohm}$



$ABCD = ABCD_{cap} ABCD_{line}$

$$ABCD_{cap} := \begin{pmatrix} 1 & -iX_c \\ 0 & 1 \end{pmatrix}$$

$$ABCD := ABCD_{cap} \cdot ABCD_{line}$$

$$ABCD = \begin{pmatrix} 0.994 + 6.564i \times 10^{-3} & 4.93 + 76.721i \\ -5.437 \times 10^{-6} + 2.42i \times 10^{-3} & 0.819 + 6.17i \times 10^{-3} \end{pmatrix}$$

Step 3 Performance of compensated line

Full and no load ABCD matrix

$$\begin{array}{ll} \underline{A} := ABCD_{1,1} & \underline{B} := ABCD_{1,2} \\ \underline{C} := ABCD_{2,1} & \underline{D} := ABCD_{2,2} \end{array}$$

Step 3A Full Load Conditions

$$\underline{V}_{Rfl} := \frac{500000}{\sqrt{3}} \quad \text{V} \quad \underline{I}_{Rfl} := \underline{I}_R \quad \underline{I}_R = 692.82 - 22.4719i$$

$$\underline{V}_s := \underline{A} \cdot \underline{V}_{Rfl} + \underline{B} \cdot \underline{I}_{Rfl} \quad |V_s| = 3.126 \times 10^5 \quad \arg(V_s) = 9.933 \text{ deg}$$

$$\underline{I}_s := \underline{C} \cdot \underline{V}_{Rfl} + \underline{D} \cdot \underline{I}_{Rfl} \quad |I_s| = 767.01 \quad \arg(I_s) = 42.303 \text{ deg}$$

$$\underline{V}_{rnl} := \frac{V_s}{A} \quad |V_{rnl}| = 3.144 \times 10^5 \quad \sqrt{3} \cdot |V_s| = 5.415 \times 10^5 \quad \text{V II}$$

$$\underline{S}_{Sfl} := 3 \cdot V_s \cdot \overline{I}_s \quad \underline{S}_{Sfl} = 6.075 \times 10^8 - 3.851i \times 10^8 \quad \text{Sending end power}$$

$$\text{Losses} \quad \underline{S}_{loss} := (V_s \cdot \overline{I}_s - \underline{V}_{Rfl} \cdot \overline{I}_{Rfl}) \cdot 3 \quad \underline{S}_{loss} = 7.548 \times 10^6 - 5.823i \times 10^8$$

Step 3b No load analysis

$$\underline{V}_{Rnl} := \frac{V_s}{A} \quad |V_{Rnl}| = 3.144 \times 10^5 \quad \sqrt{3} \cdot |V_{Rnl}| = 5.445 \times 10^5$$

$$\underline{I}_{snl} := \underline{C} \cdot \underline{V}_{Rnl} \quad |I_{snl}| = 760.688$$

Step 3c Voltage regulation

$$\underline{Reg} := \frac{(|V_{Rnl}| - |V_{Rfl}|) \cdot (100)}{|V_{Rfl}|} \quad \text{Reg} = 8.907 \quad \text{percent}$$

Step 3d Loadability at $\delta=35$ deg Remember, we use rated values for voltages

$$V_s := \frac{500000}{\sqrt{3}} \quad V_r := \frac{500000}{\sqrt{3}}$$

$$P_{\max} := \left[|V_s| \cdot \frac{|V_r| \cdot \cos\left(\arg(B) - 90 \cdot \frac{3.14}{180}\right)}{|B|} - (|V_r|)^2 \cdot \operatorname{Re}\left(\frac{A}{B}\right) \right] \cdot 3 \quad P_{\max} = 3.017 \times 10^9$$

$$P_{\max\text{prac}} := \left[|V_s| \cdot \frac{|V_r| \cdot \cos\left(\arg(B) - 35 \cdot \frac{3.14}{180}\right)}{|B|} - (|V_r|)^2 \cdot \operatorname{Re}\left(\frac{A}{B}\right) \right] \cdot 3 \quad P_{\max\text{prac}} = 1.803 \times 10^9$$

Regulation and Loadability OK

Sending end reactive power at full load is bad; no load is bad as well

Shunt Compensation

Reduce overall shunt charging current

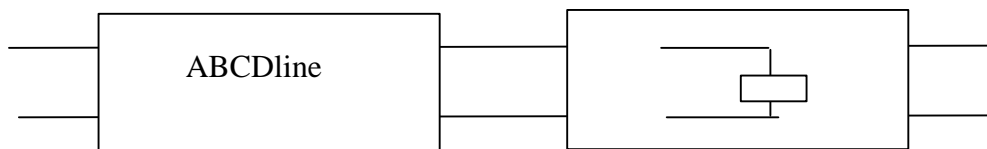
Reduce no load voltage rise

Definition

% Compensation = $100 B_r/B_c$

Connection

- One end or other
- Middle
- Split each end
- Often dictated by substation availability



$$ABCD = ABCD_{\text{line}} \quad ABCD_r$$

Example

Receiving end 50 %

$$Br := 0.5 \cdot \text{Im}(Y) \quad Br = 1.288 \times 10^{-3}$$

$$ABCD_r := \begin{pmatrix} 1 & 0 \\ -i \cdot Br & 1 \end{pmatrix} \quad \underline{ABCD} := ABCD_{line} \cdot ABCD_r$$

$$ABCD = \begin{pmatrix} 0.994 + 3.939i \times 10^{-4} & 4.483 + 136.06i \\ 2.513 \times 10^{-6} + 1.364i \times 10^{-3} & 0.819 + 6.17i \times 10^{-3} \end{pmatrix}$$

Step 3 Performance of compensated line

Full and no load ABCD matrix

$$\underline{A} := ABCD_{1,1} \quad \underline{B} := ABCD_{1,2}$$

$$\underline{C} := ABCD_{2,1} \quad \underline{D} := ABCD_{2,2}$$

Step 3A Full Load Conditions

$$\underline{VRfl} := \frac{500000}{\sqrt{3}} \quad V \quad \underline{IRfl} := IR \quad IR = 692.82 - 22.4719i$$

$$\underline{Vs} := A \cdot \underline{VRfl} + B \cdot \underline{IRfl} \quad |Vs| = 3.344 \times 10^5 \quad 1 - n \quad \arg(Vs) = 16.21 \text{ deg}$$

$$\underline{Is} := C \cdot \underline{VRfl} + D \cdot \underline{IRfl} \quad |Is| = 607.604 \quad \arg(Is) = 20.38 \text{ deg}$$

$$\underline{SSfl} := 3 \cdot \underline{Vs} \cdot \overline{Is} \quad SSfl = 6.08 \times 10^8 - 4.433i \times 10^7 \quad \text{Sending end power}$$

Losses

$$\underline{Sloss} := (\underline{Vs} \cdot \overline{Is} - \underline{VRfl} \cdot \overline{IRfl}) \cdot 3 \quad Sloss = 7.981 \times 10^6 - 2.415i \times 10^8$$

Step 3b No load analysis

$$\underline{VRnl} := \frac{Vs}{A} \quad |VRnl| = 3.363 \times 10^5 \quad \sqrt{3} \cdot |VRnl| = 5.445 \times 10^5$$

$$\underline{Isn1} := C \cdot \underline{VRnl} \quad |Isn1| = 458.844$$

Step 3c Voltage regulation

$$\underline{Reg} := \frac{(|VRnl| - |VRfl|) \cdot 100}{|VRfl|} \quad Reg = 16.508 \quad \text{percent}$$

Step 3d Loadability at $\delta=35$ deg Remember, we use rated values for voltages

$$\underline{V}_s := \frac{500000}{\sqrt{3}} \quad \underline{V}_r := \frac{500000}{\sqrt{3}}$$

$$\underline{P}_{\max} := \left[|\underline{V}_s| \cdot \frac{|\underline{V}_r| \cdot \cos\left(\arg(\underline{B}) - 90 \cdot \frac{3.14}{180}\right)}{|\underline{B}|} - (|\underline{V}_r|)^2 \cdot \operatorname{Re}\left(\frac{\underline{A}}{\underline{B}}\right) \right] \cdot 3 \quad P_{\max} = 1.775 \times 10^9$$

$$\underline{P}_{\max} := \left[|\underline{V}_s| \cdot \frac{|\underline{V}_r| \cdot \cos\left(\arg(\underline{B}) - 35 \cdot \frac{3.14}{180}\right)}{|\underline{B}|} - (|\underline{V}_r|)^2 \cdot \operatorname{Re}\left(\frac{\underline{A}}{\underline{B}}\right) \right] \cdot 3 \quad P_{\max} = 1.041 \times 10^9$$

Regulation Improved a little; Loadability did not

Sending end reactive power at full load is good

Final Example

- A. 30 % series compensation at each end
- B. 50 % shunt compensation at sending end
- C. 50 % SWITCHED (no load) compensation at receiving end



Sending and Receiving end 30 % series compensation

$$\underline{X}_c := 0.3 \cdot \text{Im}(Z) \quad X_c = 43.47 \quad \text{ohm}$$

$$\text{ABCD}_c := \begin{pmatrix} 1 & -i \cdot X_c \\ 0 & 1 \end{pmatrix}$$

Sending and Receiving end 50 % Shunt Compensation

$$\underline{B}_r := 0.5 \cdot \text{Im}(Y) \quad B_r = 1.288 \times 10^{-3}$$

$$\text{ABCD}_r := \begin{pmatrix} 1 & 0 \\ -i \cdot B_r & 1 \end{pmatrix}$$

Step 3 Performance of compensated line

Full and no load ABCD matrix

Step 3A Full Load Conditions

$$ABCD_FL := ABCDr \cdot ABCDc \cdot ABCDline \cdot ABCDc$$

$$ABCD_FL = \begin{pmatrix} 0.924 + 6.406i \times 10^{-3} & 5.029 + 60.281i \\ 2.818 \times 10^{-6} + 1.229i \times 10^{-3} & 1.002 - 7.417i \times 10^{-5} \end{pmatrix}$$

$$\begin{aligned} \underline{A} &:= ABCD_FL_{1,1} & \underline{B} &:= ABCD_FL_{1,2} \\ \underline{C} &:= ABCD_FL_{2,1} & \underline{D} &:= ABCD_FL_{2,2} \end{aligned}$$

$$\underline{VRfl} := \frac{500000}{\sqrt{3}} \quad \text{V} \quad \underline{IRfl} := IR \quad IR = 692.82 - 22A719i$$

$$\underline{Vs} := A \cdot \underline{VRfl} + B \cdot \underline{IRfl} \quad |Vs| = 2.872 \times 10^5 \quad 1 - n \quad \arg(Vs) = 8.505 \text{ deg}$$

$$\underline{Is} := C \cdot \underline{VRfl} + D \cdot \underline{IRfl} \quad |Is| = 706.34 \quad \arg(Is) = 10.318 \text{ deg}$$

$$\sqrt{3} \cdot |Vs| = 4.974 \times 10^5 \quad \text{VLL}$$

$$\underline{SSfl} := 3 \cdot \underline{Vs} \cdot \overline{\underline{Is}} \quad SSfl = 6.082 \times 10^8 - 1.925i \times 10^7 \quad \text{Sending end power}$$

Sending end reactive power is 19 MVAR leading

$$\text{Losses} \quad \underline{Sloss} := (\underline{Vs} \cdot \overline{\underline{Is}} - \underline{VRfl} \cdot \overline{\underline{IRfl}}) \cdot 3 \quad Sloss = 8.204 \times 10^6 - 2.165i \times 10^8$$

Step 3b No load analysis

We now switch in the no-load reactor

$$ABCD_NL := ABCDr \cdot ABCDc \cdot ABCDline \cdot ABCDc \cdot ABCDr$$

$$ABCD_NL = \begin{pmatrix} 1.002 - 7.417i \times 10^{-5} & 5.029 + 60.281i \\ 2.722 \times 10^{-6} - 6.218i \times 10^{-5} & 1.002 - 7.417i \times 10^{-5} \end{pmatrix}$$

$$\begin{aligned} \underline{A} &:= ABCD_NL_{1,1} & \underline{B} &:= ABCD_NL_{1,2} \\ \underline{C} &:= ABCD_NL_{2,1} & \underline{D} &:= ABCD_NL_{2,2} \end{aligned}$$

$$\underline{VRnl} := \frac{Vs}{A} \quad |VRnl| = 2.866 \times 10^5 \quad \sqrt{3} \cdot |Vrnl| = 5.445 \times 10^5$$

$$\underline{Isn1} := C \cdot VRnl \quad |Isn1| = 17.838$$

Step 3c Voltage regulation

$$\underline{Reg} := \frac{(|VRnl| - |VRfl|) \cdot (100)}{|VRfl|} \quad Reg = -0.71 \quad \text{percent}$$

Step 3d Loadability at $\delta=35$ deg Remember, we use rated values for voltages

$$\begin{aligned} \underline{A} &:= ABCD_FL_{1,1} & \underline{B} &:= ABCD_FL_{1,2} \\ \underline{C} &:= ABCD_FL_{2,1} & \underline{D} &:= ABCD_FL_{2,2} \end{aligned}$$

$$\underline{Pmax} := \left[|Vs| \cdot \frac{|Vr| \cdot \cos\left(\arg(B) - 35 \cdot \frac{3.14}{180}\right)}{|B|} - (|Vr|)^2 \cdot \operatorname{Re}\left(\frac{A}{B}\right) \right] \cdot 3 \quad Pmax = 2.285 \times 10^9$$

