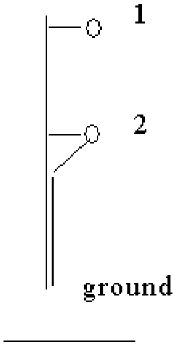
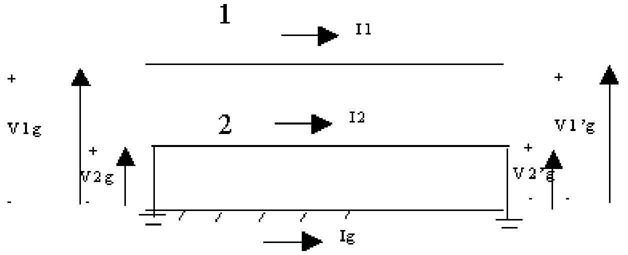
An application of Carson's equations

Example: Consider a single phase line 1 mile long, Linnet conductor, vertical configuration at 5' spacing. The hot conductor carries 100 A.The neutral is solidly grounded at each end(what does this mean ?) Compute impedances, voltage drop, neutral current and ground current. Use the simplified Carson Equations





$$j := \sqrt{-1}$$

$$Z_{1,1} := 0.306 + 0.0953 + j \cdot 0.1213 \cdot \left(\ln\left(\frac{1}{0.0244} \right) + 7.934 \right) \qquad Z_{1,1} = 0.401 + 1.413i$$

$$Z_{2,2} := Z_{1,1}$$

$$Z_{1,2} := 0.0 + 0.0953 + j \cdot 0.1213 \cdot \left(\ln\left(\frac{1}{5} \right) + 7.934 \right) \qquad Z_{1,2} = 0.095 + 0.767i$$

$$ZP := \begin{pmatrix} Z_{1,1} & Z_{1,2} \\ Z_{1,2} & Z_{2,2} \end{pmatrix} \qquad ZP = \begin{pmatrix} 0.401 + 1.413i & 0.095 + 0.767i \\ 0.095 + 0.767i & 0.401 + 1.413i \end{pmatrix}$$

The basic equations are

V1g-V1'g = Z11 I1 + Z12 I2
V2g-V2'g= Z21 I1 + Z22 I2

V2g=0 and V2'g=0

So I2= -Z21 I1/Z22

and

V1g-V1'g = (Z11-Z12Z21/Z22)I1

Line impedance =(Z11-Z12Z21/Z22)

$I1 := 100$

$I2 := -I1 \cdot \frac{Z_{1,2}}{Z_{2,2}}$ $\quad |I2| = 52.636$ $\quad \arg(I2) = -171.224\,\text{deg}$

$Iearth := -I1 - I2$ $\quad |Iearth| = 48.647$ $\quad \arg(Iearth) = 170.498\,\text{deg}$

$Zp := Z_{1,1} - \frac{Z_{1,2} \cdot Z_{1,2}}{Z_{2,2}}$ $\quad Zp = 0.413 + 1.006i$

$Vd1 := Zp \cdot I1$ $\quad |Vd1| = 108.766$

Our conventional formula ignores the earth path and gives us an impedance appropriate for an ungrounded neutral. I will call it Zp1p

$Zp1p := \left(0.306 + j \cdot 0.1213 \cdot \ln\left(\frac{5}{.0244}\right)\right) \cdot 2$ $\quad Zp1p = 0.612 + 1.291i$

This is often called the loop formula It follows from Carson's formula by setting I1=I2

Capacitance Calculation

In lecture 1 we had described the general potential coefficients model for a system of conductors

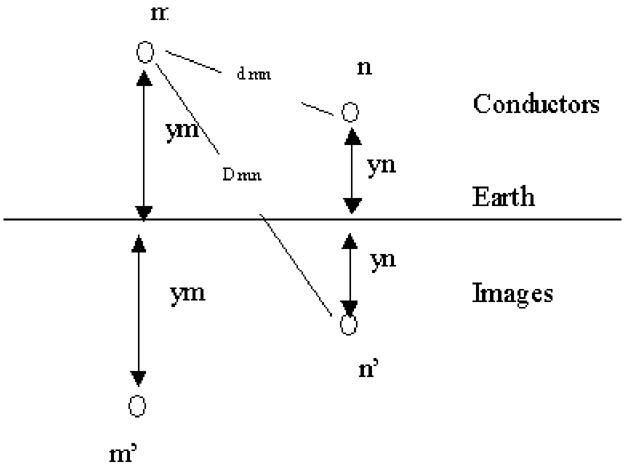
The vector of voltages with respect to reference(ground) as a function of the vector of charges(C/m) on the conductors is given by

$V = P q$

The elements of the potential coeffecient matrix are given by

$$P_{mn} = (1/2\pi\varepsilon_o) \ln(D_{mn}/d_{mn}) \qquad \text{V/C/m}$$

where Dmn are conductor to image distances and dmn are conductor to conductor distances



The primitive capacitance matrix is the inverse of P

$C = P^{-1}$

The phase potential coefficient matrix is obtained by eliminating shield wires in exactly the same way as was done for the impedance matrix. The inverse of this phase-potential coefficient matrix is the phase capacitance matrix. Sequence capacitances can be calculated as usual using the symmetrical component transformation.

The calculations are illustrated in what follows. The lines are the same as in Lecture 3

Examample 1 AIP1A with corncrake conductor (Same as Lecture 3)

$\text{Conductors} := 5$

Coordinates Conductors are numbered 1,2,3; shield wires 4 and 5 $\frac{3}{192} = 0.016$

$x_1 := 0$ $x_4 := 10$ $x_2 := 23.5$ $x_5 := 37$ $x_3 := 47$

$y_1 := 42$ $y_4 := 74$ $y_2 := 42$ $y_5 := 72$ $y_3 := 42$ feet

Phases are Corncrake $\text{radius} := \frac{1.2}{24}$ feet

$Ds_1 := \sqrt{\text{radius} \cdot 1.5}$ $Ds_2 := \sqrt{\text{radius} \cdot 1.5}$ $Ds_3 := \sqrt{\text{radius} \cdot 1.5}$ feet

shield is 3/8 in steel $\text{radsteel} := 0.016$ $Ds_4 := \text{radsteel}$ $Ds_5 := \text{radsteel}$ feet

Conductor to Conductor distance

$$D(m,n) := \left| \begin{array}{l} Ds_m \ \text{ if } m = n \\ \sqrt{\left(x_m - x_n\right)^2 + \left(y_m - y_n\right)^2} \ \text{ otherwise} \end{array} \right.$$

Conductor to image distance

$$H(m,n) := \left| \begin{array}{l} 2 \cdot y_m \ \text{ if } m = n \\ \sqrt{\left(x_m - x_n\right)^2 + \left(y_m + y_n\right)^2} \ \text{ otherwise} \end{array} \right.$$

distance matrix

$m := 1, 2 .. 5$ $n := 1, 2 .. 5$

$d_{m,n} := D(m,n)$

$h_{m,n} := H(m,n)$

$$d = \begin{pmatrix} 0.274 & 23.5 & 47 & 33.526 & 47.634 \\ 23.5 & 0.274 & 23.5 & 34.731 & 32.898 \\ 47 & 23.5 & 0.274 & 48.918 & 31.623 \\ 33.526 & 34.731 & 48.918 & 0.016 & 27.074 \\ 47.634 & 32.898 & 31.623 & 27.074 & 0.016 \end{pmatrix}$$

Potential Coefficient matrix $\varepsilon_o := 8.854 \cdot 10^{-12}$

$$P_{m,n} := \left(\frac{1}{2 \cdot \pi \cdot \varepsilon_o}\right) \cdot \ln\left(\frac{h_{m,n}}{d_{m,n}}\right)$$

$$P = \begin{pmatrix} 1.029 \times 10^{11} & 2.357 \times 10^{10} & 1.289 \times 10^{10} & 2.238 \times 10^{10} & 1.659 \times 10^{10} \\ 2.357 \times 10^{10} & 1.029 \times 10^{11} & 2.357 \times 10^{10} & 2.18 \times 10^{10} & 2.247 \times 10^{10} \\ 1.289 \times 10^{10} & 2.357 \times 10^{10} & 1.029 \times 10^{11} & 1.639 \times 10^{10} & 2.312 \times 10^{10} \\ 2.238 \times 10^{10} & 2.18 \times 10^{10} & 1.639 \times 10^{10} & 1.642 \times 10^{11} & 3.059 \times 10^{10} \\ 1.659 \times 10^{10} & 2.247 \times 10^{10} & 2.312 \times 10^{10} & 3.059 \times 10^{10} & 1.637 \times 10^{11} \end{pmatrix}$$

Capacitance matrix $C := P^{-1}$

$$C = \begin{pmatrix} 1.054 \times 10^{-11} & -1.948 \times 10^{-12} & -5.929 \times 10^{-13} & -1.021 \times 10^{-12} & -5.264 \times 10^{-13} \\ -1.948 \times 10^{-12} & 1.097 \times 10^{-11} & -1.939 \times 10^{-12} & -8.343 \times 10^{-13} & -8.791 \times 10^{-13} \\ -5.929 \times 10^{-13} & -1.939 \times 10^{-12} & 1.056 \times 10^{-11} & -5.166 \times 10^{-13} & -1.068 \times 10^{-12} \\ -1.021 \times 10^{-12} & -8.343 \times 10^{-13} & -5.166 \times 10^{-13} & 6.568 \times 10^{-12} & -9.366 \times 10^{-13} \\ -5.264 \times 10^{-13} & -8.791 \times 10^{-13} & -1.068 \times 10^{-12} & -9.366 \times 10^{-13} & 6.61 \times 10^{-12} \end{pmatrix} \quad \frac{F}{m}$$

Phase Capacitance Matrux

$\mathrm{Ppp} := \mathrm{submatrix}(P, 1, 3, 1, 3)$

$\mathrm{Pps} := \mathrm{submatrix}(P, 4, 5, 1, 3)$

$\mathrm{Pss} := \mathrm{submatrix}(P, 4, 5, 4, 5)$

$\mathrm{Pp} := \mathrm{Ppp} - \mathrm{Pps}^{T} \cdot \mathrm{Pss}^{-1} \cdot \mathrm{Pps}$

$\mathrm{Cp} := \mathrm{Pp}^{-1}$

$$\mathrm{Cp} = \begin{pmatrix} 1.054 \times 10^{-11} & -1.948 \times 10^{-12} & -5.929 \times 10^{-13} \\ -1.948 \times 10^{-12} & 1.097 \times 10^{-11} & -1.939 \times 10^{-12} \\ -5.929 \times 10^{-13} & -1.939 \times 10^{-12} & 1.056 \times 10^{-11} \end{pmatrix}$$

Sequence Capacitances

Symmetrical Component Tranformation

$$A := \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{i \cdot -120\mathrm{deg}} & e^{(i \cdot 120\mathrm{deg})} \\ 1 & e^{(i \cdot 120\mathrm{deg})} & e^{i \cdot -120\mathrm{deg}} \end{bmatrix} \qquad A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -0.5 - 0.866i & -0.5 + 0.866i \\ 1 & -0.5 + 0.866i & -0.5 - 0.866i \end{pmatrix}$$

$\mathrm{Cs} := A^{-1} \cdot \mathrm{Cp} \cdot A$

$$\mathrm{Cs} = \begin{pmatrix} 7.705 \times 10^{-12} & 1.489 \times 10^{-13} + 2.704i \times 10^{-13} & 1.489 \times 10^{-13} - 2.704i \times 10^{-13} \\ 1.489 \times 10^{-13} - 2.704i \times 10^{-13} & 1.218 \times 10^{-11} & -5.203 \times 10^{-13} - 9.03i \times 10^{-13} \\ 1.489 \times 10^{-13} + 2.704i \times 10^{-13} & -5.203 \times 10^{-13} + 9.03i \times 10^{-13} & 1.218 \times 10^{-11} \end{pmatrix}$$

Using the transposed line calculation with earth and shield ignored, we get the following value for postive sequence capacitance

$od := 1.2$ in $r := \frac{od}{24}$ $r = 0.05$ feet Bundle $d := 1.5$ feet $\varepsilon_o := 8.854 \cdot 10^{-12}$

$Dsc := \sqrt{r \cdot d}$ $Dsc = 0.274$ feet

$Deq := \sqrt[3]{23.5 \cdot 23.5 \cdot 47}$ $Deq = 29.608$

$$C1 := \frac{2 \cdot \pi \cdot \varepsilon_o}{\ln\left(\frac{Deq}{Dsc}\right)} \qquad C1 = 1.188 \times 10^{-11} \ \frac{F}{m}$$

$$C1 \cdot 1609 = 1.911 \times 10^{-8} \ \frac{F}{mi}$$

Example 2

The line is 200 miles long. The sending end is energized at 345 kV, receiving end is open. Find the charging current

Charging admittance

$$Yp := j \cdot 377 \cdot 1609 \cdot 200 Cp$$

$$Vp := \begin{pmatrix} 200 \\ 200 \cdot e^{i \cdot -120deg} \\ 200 \cdot e^{i \cdot 120deg} \end{pmatrix}$$

$$Ip := Yp \cdot Vp$$

$$Ip = \begin{pmatrix} -0.028 + 0.287i \\ 0.271 - 0.157i \\ -0.263 - 0.119i \end{pmatrix}$$

$\left|Ip_1\right| = 0.288$ kA

$\left|Ip_2\right| = 0.313$

$\left|Ip_3\right| = 0.288$

Example 3 A 345 kV and a 115 kV line sharing a right of way(Same as Lecture 3)

First Line (345 kv)

$\text{Conductors} := 5$

Coordinates Conductors are numbered 1,2,3; shield wires 7 and 8

$x_1 := 0 \quad x_7 := 10 \quad x_2 := 23.5 \quad x_8 := 37 \quad x_3 := 47$

$y_1 := 42 \quad y_7 := 74 \quad y_2 := 42 \quad y_8 := 72 \quad y_3 := 42$ feet

Phases are Corncrake $\text{radius} := \frac{1.2}{24}$

Equivalent conductor for bundle

$Ds_1 := \sqrt{\text{radius} \cdot 1.5} \quad Ds_2 := \sqrt{\text{radius} \cdot 1.5} \quad Ds_3 := \sqrt{\text{radius} \cdot 1.5}$ feet

shield is 3/8 in steel $\text{radsteel} := 0.016 \quad Ds_7 := \text{radsteel} \quad Ds_8 := \text{radsteel}$ feet

Line 2 (115 kv)

Coordinates Conductors are numbered 4,5,6; shield wires 9 and 10

$x_4 := -115.5 \quad x_9 := -109.5 \quad x_5 := -101.5 \quad x_{10} := -93.5 \quad x_6 := -87.5$

$y_4 := 32.7 \quad y_9 := 43.75 \quad y_5 := 32.7 \quad y_{10} := 72 \quad y_6 := 43.75$ feet

Phases are Hawk $\text{radius} := \frac{.858}{24}$ feet

$Ds_4 := \text{radius} \quad Ds_5 := \text{radius} \quad Ds_6 := \text{radius}$ feet

shield is 3/8 in steel $\text{radsteel} := 0.012 \quad Ds_9 := \text{radsteel} \quad Ds_{10} := \text{radsteel}$ feet

Conductor to Conductor distance

$$D(m,n) := \left| \begin{array}{l} Ds_m \text{ if } m = n \\ \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2} \text{ otherwise} \end{array} \right.$$

Conductor to image distance

$$H(m,n) := \left| \begin{array}{l} 2 \cdot y_m \text{ if } m = n \\ \sqrt{(x_m - x_n)^2 + (y_m + y_n)^2} \text{ otherwise} \end{array} \right.$$

$\sqrt{(x_m - x_n) + (y_m + y_n)}$ otherwise

distance matrix

$m := 1, 2 .. 10 \quad n := 1, 2 .. 10$

$d_{m,n} := D(m,n)$

$h_{m,n} := H(m,n)$

Potential Coefficient matrix $\qquad \varepsilon_o := 8.854 \cdot 10^{-12}$

$$P_{m,n} := \left(\frac{1}{2 \cdot \pi \cdot \varepsilon_o}\right) \cdot \ln\left(\frac{h_{m,n}}{d_{m,n}}\right) \qquad C := P^{-1}$$

$C \cdot 10^{12} =$

	1	2	3	4	5	6	7	8	9
1	10.572	-1.935	-0.584	-0.063	-0.095	-0.23	-0.998	-0.514	-0.088
2	-1.935	10.98	-1.936	-0.028	-0.038	-0.091	-0.824	-0.873	-0.04
3	-0.584	-1.936	10.559	-0.021	-0.024	-0.055	-0.51	-1.064	-0.028
4	-0.063	-0.028	-0.021	8.081	-1.242	-0.495	-0.053	-0.032	-1.265
5	-0.095	-0.038	-0.024	-1.242	8.193	-1.106	-0.067	-0.036	-1.1
6	-0.23	-0.091	-0.055	-0.495	-1.106	7.717	-0.156	-0.082	-0.77
7	-0.998	-0.824	-0.51	-0.053	-0.067	-0.156	6.586	-0.926	-0.075
8	-0.514	-0.873	-1.064	-0.032	-0.036	-0.082	-0.926	6.616	-0.044
9	-0.088	-0.04	-0.028	-1.265	-1.1	-0.77	-0.075	-0.044	6.94
10	-0.239	-0.111	-0.08	-0.363	-0.4	-0.851	-0.217	-0.125	-0.591

$\frac{F}{m}$

Phase Capacitance Matrux

$Ppp := \mathrm{submatrix}(P, 1, 6, 1, 6)$

$Pps := \mathrm{submatrix}(P, 7, 10, 1, 6)$

$Pss := \mathrm{submatrix}(P, 7, 10, 7, 10)$

$$Pp := Ppp - Pps^{T} \cdot Pss^{-1} \cdot Pps$$

$$Cp := Pp^{-1}$$

$$Cp \cdot 10^{12} = \begin{pmatrix} 10.572 & -1.935 & -0.584 & -0.063 & -0.095 & -0.23 \\ -1.935 & 10.98 & -1.936 & -0.028 & -0.038 & -0.091 \\ -0.584 & -1.936 & 10.559 & -0.021 & -0.024 & -0.055 \\ -0.063 & -0.028 & -0.021 & 8.081 & -1.242 & -0.495 \\ -0.095 & -0.038 & -0.024 & -1.242 & 8.193 & -1.106 \\ -0.23 & -0.091 & -0.055 & -0.495 & -1.106 & 7.717 \end{pmatrix} \qquad \frac{F}{m}$$

Sequence Capacitances

Symmetrical Component Tranformation

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -0.5-0.866i & -0.5+0.866i \\ 1 & -0.5+0.866i & -0.5-0.866i \end{pmatrix}$$

$$\text{CeroM} := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\text{AU} := \text{augment}(A, \text{CeroM})$ $\text{AL} := \text{augment}(\text{CeroM}, A)$

$\text{A2} := \text{stack}(\text{AU}, \text{AL})$

$\text{Cs} := \text{A2}^{-1} \cdot \text{Cp} \cdot \text{A2}$

$$\text{Re}(\text{Cs}) \cdot 10^{12} = \begin{pmatrix} 7.733 & 0.159 & 0.159 & -0.215 & 0.052 & 0.052 \\ 0.159 & 12.189 & -0.517 & -0.086 & 0.029 & 0.019 \\ 0.159 & -0.517 & 12.189 & -0.086 & 0.019 & 0.029 \\ -0.215 & -0.086 & -0.086 & 6.101 & 0.121 & 0.121 \\ 0.052 & 0.029 & 0.019 & 0.121 & 8.945 & -0.117 \\ 0.052 & 0.019 & 0.029 & 0.121 & -0.117 & 8.945 \end{pmatrix}$$

$$\text{Im}(\text{Cs}) \cdot 10^{12} = \begin{pmatrix} 0 & 0.268 & -0.268 & 0 & -0.063 & 0.063 \\ -0.268 & 0 & -0.902 & -0.017 & -0.022 & 0.032 \\ 0.268 & 0.902 & 0 & 0.017 & -0.032 & 0.022 \\ 0 & 0.017 & -0.017 & 0 & 0.079 & -0.079 \\ 0.063 & 0.022 & 0.032 & -0.079 & 0 & -0.569 \\ -0.063 & -0.032 & -0.022 & 0.079 & 0.569 & 0 \end{pmatrix}$$

Note : Unsymmetrical lines will result in a "complex sequence capacitance"

Example 4

The 345 kV line is energized. The 115 kV line is open. Find the voltage induced in the phases of the open line

Let Ip1, Vp1 represent the vector of phase currents and voltages and voltages for the 345 kV line and Ip2, Vp2 represent the vector of phase currents and voltages and voltages for the 115 kV kine

The relationship I = jωC V can be partitioned as

$$\begin{pmatrix} \text{Ip1} \\ \text{Ip2} \end{pmatrix} := j \cdot \omega \cdot \begin{pmatrix} \text{C11} & \text{C12} \\ \text{C21} & \text{C22} \end{pmatrix} \cdot \begin{pmatrix} \text{Vp1} \\ \text{Vp2} \end{pmatrix}$$

sincwe Ip2=0

$0 = j\omega C21\ V1 + j\omega C22\ V2$

$V2 = -C22^{-1}\ C21\ V1$

$C22 := \text{submatrix}(Cp, 4, 6, 4, 6)$ $\qquad$ $C21 := (\text{submatrix}(Cp, 4, 6, 1, 3))$

$$V1 := \begin{pmatrix} 200 \\ 200 \cdot e^{i \cdot -120deg} \\ 200 \cdot e^{i \cdot 120deg} \end{pmatrix}$$

$V2 := -C22^{-1} \cdot C21 \cdot V1$

$$V2 = \begin{pmatrix} 1.588 - 0.29i \\ 2.401 - 0.459i \\ 4.513 - 0.882i \end{pmatrix} \qquad kV$$